VU VRIJE UNIVERSITEIT AMSTERDAM — School of Business and Economics

Fiscal Futures: Finding Optimal Fiscal Policy Rules under Macroeconomic Uncertainty

Anne Li Cnossen

(2720503)

September 20, 2023

Thesis MSc EOR - Operations Research

Thesis committee:
Prof. dr. Bernd Heidergott (supervisor)
Dr. Vinzenz Ziesemer (co-reader)

Governments mostly employ static constraints on fiscal policy to control government debt levels, such as the Stability and Growth Pact in the European Union and the debt limit in the United States. Static constraints do not react to changes in the macroeconomic environment and have proven to be unhelpful during recent periods of macroeconomic instability. This thesis presents a method for evaluating fiscal rules – both static and dynamic – in a wide range of realistic economic scenarios. An (stochastic) optimisation method is introduced to select a combination of fiscal rules that optimally provides fiscal stability while minimising the probability of default. This thesis applies the optimisation method to classic fiscal rules from the literature and finds that a combination of dynamic fiscal rules is optimal. Moreover, it was found that the introduction of static constraints from the Stability and Growth Pact leads to fiscal instability.

Keywords: Fiscal policy, debt sustainability, stochastic optimisation

1 Introduction

The government plays a pivotal role in shaping a country's economy, intervening through its expenditure and revenue policies. Spending involves the use of public funds to purchase goods and services, such as education, healthcare, and welfare, by (local) government bodies, while the government revenue mainly comprises taxes and social contributions paid by households and businesses (interest payments not included). The disparity between revenue and expenditure (excluding interest payments) is known as the primary balance, and if spending exceeds revenue, it is often referred to as the primary deficit of a country. Over the course of history, government spending and revenue have exhibited significant volatility due to economic, political, and financial factors. Besides affecting the economy in the short and long run, government finances also influence the level of government debt: primary deficits increase public debt, while primary surpluses reduce debt. In addition, elevated debt levels are accompanied by additional costs to service interest payments. Therefore, determining how much to spend and how to allocate funds is a challenging, yet essential, task of the government. Here, fiscal policy comes into place, which is mainly concerned with determining government expenditure and revenue over time.

Effective management of public debt policy is crucial to ensuring long-term fiscal sustainability, stability, and economic growth. Fiscal policy involves the strategic use of government spending, taxation, and borrowing to achieve (predetermined) economic goals. Whereas the primary purpose of fiscal policy is to stabilise and promote economic growth, it also addresses various other (economic) challenges such as redistribution of income and mitigating climate change. Governments predominantly rely on static regulations to manage fiscal policy and control government debt levels, exemplified by the Stability and Growth Pact in the European Union and the debt limit in the United States. However, these constraints are rigid and do not adapt to changes in the macroeconomic environment.

Current fiscal policy lacks long-term foresight and causes unnecessary volatility and harm to the economy. Despite the central role of fiscal policy in economic stability, recent history has revealed its inherent volatility in the Netherlands. In the aftermath of the Financial Crisis and the European debt crisis, policymakers focused predominantly on reducing elevated debt ratios, a strategy that later proved detrimental to economic growth. More recently, the general policy convention appears to have turned around with primary balances predominantly characterised by large budget deficits, implying increased government spending and/or reduced taxes (Jacobs, 2015; Centraal Bureau voor de Statistiek, 2022; Khan, 2019; Leeuw and Bruns, 2022). The current inconsistent fiscal policy had led to suboptimal policy outcomes and caused instability in the public sector, as departments had to constantly adjust to fluctuating budgets. This has imposed various long-term repercussions on society, such as the current deep-rooted issues within the tax department, the national housing crisis, and infrastructure problems due to overdue maintenance (Lukkezen, 2021; Vries, 2020; Wijnbergen, 2021; Algemene Rekenkamer, 2021).

In addition, it has been observed that fiscal policy was often procyclical (50%) or neutral ($25-35\%$) (Homan and Suyker, 2015). While countercyclical policy can alleviate the impact of economic shocks and dampen the business cycle, procyclical policy tends to exacerbate economic recessions (Jacobs, 2015). Imposing additional fiscal contractions in periods of economic downturn inhibits economic recovery. Especially in the presence of (positive) fiscal multipliers – a measure used to illustrate the impact of fiscal policy on the economy – it is beneficial to impose countercyclical policy. According to Katz and Bettendorf (2023), the impact of fiscal policy on the economy is even more pronounced during times of low growth, making investments to stimulate growth (i.e., increase government spending) relatively inexpensive. Moreover, academic research emphasises the importance of the permanent effects of recessions on productivity due to a loss of knowledge and trained workforce, lagging R&D and capital accumulation and worsening financing conditions (Cerra et al., 2023). The existence of hysteresis implies that prudent fiscal policy is imperative, such that recessions can be reduces.

Additionally, the evaluation of fiscal policy leaves much to be desired. The current indicator of debt sustainability, *the long-term sustainability gap indicator*, tries to determine the balance between future expected government expenditure and revenue (European Commission, 2017). However, in this analysis, they fail to incorporate macroeconomic uncertainty in growth, interest, and the primary balance. Furthermore, the model uses an artificially elevated interest rate, which should act as a risk premium. However, this premium lacks any theoretical foundation (Jacobs, 2020). In addition, this approach can only be used when the average interest rate is lower than the rate of growth, which has not been the case recently.

As such, it can be concluded that there is an urgent need for a reform of fiscal policy. Current academic literature on fiscal policy focusses primarily on optimising a restrictive objective function related to the optimal distribution of resources (see, for example, Barro (1979), Aiyagari et al. (2002), and Bhandari et al. (2017)), and determining the current stance of the public finances by scrutinising the relation between debt ratios and the government balances via the so-called *fiscal response functions* (e.g. Bohn (1998) and Ghosh, Kim, et al. (2013)). Research has also been conducted on evaluation measures. Recently, policy institutions have introduced *debt sustainability analysis* (DSA), which can be used to simulate debt dynamics in different scenarios (IMF, 2022; Heimberger, 2023). In contrast to many deterministic models, DSA tries to incorporate uncertainty embedded in the long-term development of the debt ratio. DSA can be used to identify risks to the sustainability of debt and inform policy makers about the appropriate responses to fiscal policy (Guzman and Heymann, 2015). Using a DSA model, different scenarios can be simulated to gain an understanding of the development of debt and the interactions between the different macroeconomic variables related to the projection of debt.

This thesis introduces a novel approach to assess fiscal regulations, encompassing both static and dynamic rules, across a wide spectrum of plausible economic scenarios. Until now, DSA has been mainly used to model the development of debt under existing fiscal policy. This research seeks to extend on this framework by utilising it to compare various (classic) fiscal policy rules. Consequently, evaluations of existing policy rules will be used to identify the optimal combination of dynamic fiscal rules. Influential research conducted by Barro (1979) showed favourable results for a neutral debt target policy, which means that governments should aim for a consistent debt ratio over time. Furthermore, recent research by Bhandari et al. (2017) proposed a slight adjustment: a gradual return towards a long-term debt goal. However, these optimisation analyses did not take into account the influence of the business cycles nor the presence of fiscal multipliers. As such, this analysis will draw upon the welfare optimising literature by incorporating empirical phenomena into the optimisation problem in order to find fiscal policy that promotes macroeconomic stability and ensures sustainable government finances. To accomplish this, drastic alterations in fiscal policy must be avoided (i.e., minimise additional fiscal contractions and/or expansions). Large fluctuations in government budgets reinforce the business cycle and cause instability in the public sector, which will resonate in the economy. Moreover, fiscal policy must also control government debt levels to remain sustainable, which can be achieved by minimising the risk of default. Lastly, stability can also be promoted by maintaining countercyclical policies, as this tends to reduce the impact of the business cycle.

Using a stochastic approximation algorithm, the aim of this paper is to find (countercylical) fiscal policy that fosters fiscal stability, while minimising the probability of default, with an emphasis on the following research question: *Does amending a neutral debt strategy with explicit debt goals and business cycle indicators produce a strategy that is more stable, countercyclical, and characterised by a low risk of default?* Based on historical data from *CPB Netherlands Bureau for Economic Policy Analysis*, this thesis seeks to optimise fiscal policy for the Netherlands using a stochastic gradient descent algorithm.

The results of the benchmark scenarios and the optimisation procedure indicate that the optimised fiscal policy is a significant improvement compared to the implementation of the static ad hoc constraints (such as the debt ratio and deficit constraint of the *Stability and Growth Pact*) and the dynamic neutral debt target. While the ad hoc constraints drastically reduce the risk of default, their implementations are also accompanied by a surge in instability. The neutral debt target, on the other hand, intervenes more gently. However, the risk of default was found to remain relatively high in the neutral debt target scenario. In contrast to the benchmark scenarios, the optimised rule enhances the stability and reduces the risk of default. However, it does not improve the countercylical efficiency, as the policy rule was found to be acylical.

The remainder of this thesis is structured as follows. First, there will be an overview of the literature on fiscal policy, debt sustainability, and welfare optimisation. Consequently, Section 3 will discuss the DSA framework in more detail and gives an overview of the optimisation algorithm employed. Section 4 consists of a detailed description of the model employed in this thesis, including a declaration of the data and assumptions used in this analysis. The next section shows the results of the benchmark scenarios and the optimisation results. Lastly, there will be a discussion of the results followed by some concluding remarks in Section 7.

2 Literature review

The academic discourse on the optimisation of fiscal policy is three-fold. It consists of literature related to the analysis of current policy, research on fiscal sustainability, and there is a vast academic field on optimising fiscal policy. This section will first give an overview of current fiscal policy analyses and the latest insight regarding debt sustainability. Consequently, we will investigate the literature on the optimisation of fiscal policy from a theoretical and empirical perspective and how this thesis contributes to the existing literature.

2.1 Analysing current policy

Initially, indicators to analyse fiscal policy were based on the accounting principle of the debt ratio. The academic literature distinguishes two different approaches to qualitatively evaluate fiscal and debt sustainability. First, the *intertemporal government budget constraint* (IGBC) evaluates the present value of government expenditures compared to the present value of revenues (Hamilton and Flavin, 1985). Using the IGBC, it can be determined what the current amount of debt should be to be equivalent to the discounted future government cash flows. A second approach was introduced by Kremers (1989), who proposed a metric to assess whether debt would explode under current and future fiscal policies. According to him, it is sufficient to prove that the debt ratio converges to a constant in the limit. These analyses relied on the government discount rate to determine the present value of revenues; however, Bohn (1998), contested this approach, suggesting that stochastic discounting should be used instead. According to him, uncertainty and risk-averse agents cause government discount rates to be inappropriate. His findings later became the backbone of the literature on fiscal response functions, one of the main pillars in the current discourse on public debt policy assessments.

Research on *fiscal response functions* (FRF) has gained insight into how governments react to fluctuations in economic conditions, with an emphasis on the relationship between debt ratios and primary balances (Berti et al., 2016). It is an evidence-based measure used to quantify the current fiscal response to an increase in debt. Initially, Bohn (1998) analysed the corrective measures taken to respond to a rise in public debt. Using a univariate regression model and controlling for wartimes and cyclical fluctuations, Bohn (1998) has found evidence for a positive relationship between primary surpluses and the debt ratio. In his paper, Bohn (1998) suggested that fiscal policy can be evaluated merely based on the sign of the debt coefficient, regardless of the size. This is based on the assumption that a positive coefficient would eventually lead to mean reversion. A positive correlation between primary balance and increased debt suggests that governments take action to limit the growth of debt by implementing fiscal consolidation when debt levels rise. Subsequent studies, which have employed not only linear functions (e.g., Baldi and Staehr (2016), Weichenrieder and Zimmer (2014), and Zedda et al. (2011)) but also more complex nonlinear functions (e.g., Fournier and Fall (2015), Lukkezen and Rojas-Romagosa (2013), and Lukkezen, Rojas-Romagosa, et al. (2012)), have confirmed the significant positive correlation between primary surpluses and debt ratios. However, they suggest that the size of the coefficient is decisive for the evaluation of debt (Fournier and Fall, 2015; Ghosh, Ostry, et al., 2013; Ghosh, Kim, et al., 2013).

Academic research on FRF has greatly contributed to the understanding of historical fiscal policy behaviour. Extensive research was performed to distinguish fiscal response functions for individual countries. Berti et al. (2016) compared the estimates of different studies and found a median debt coefficient of 0.05 for the Netherlands, with a maximum slightly above 0.10 and a minimum around 0.01. Based on their research, it can be concluded that there is a consensus on the sign of the Dutch debt coefficient, which means that historically the Netherlands was characterised by a tendency to adjust its fiscal policy positively in response to a rise in debt ratio.

2.2 Debt sustainability

It is essential to not only evaluate the current financial state, but also consider future fiscal practises and determine whether financial commitments will remain achievable in the foreseeable future. Analysing the sustainability of debt involves assessing a country's ability to meet its debt obligations without jeopardising its long-term fiscal and economic stability. For government debt to be sustainable, it must be solvent and liquid (Bouabdallah et al., 2017). *Solvency* evaluates the medium- to long-term sustainability of government debt by examining whether future primary surpluses are sufficient to repay outstanding debt. This requires that the government budget constraint is fulfilled, i.e., the net present value (NPV) of future balances must at least equal the NPV of outstanding debt. *Liquidity* on the other hand, evaluates the short-term ability to service upcoming obligations by assessing a governments capacity to access the financial market and its cash flow management.

The assessment of debt sustainability relies on the definition of sustainable debt. The European Commission (EC), the International Monetary Fund (IMF) and the European Central Bank (ECB) evaluate the sustainability of debt by assessing the solvency of the public sector, which means that the maintenance of (unchanged) future primary balances must be sufficient to repay outstanding debt (Bouabdallah et al., 2017; Heimberger, 2023; Guzman and Heymann, 2015). Blanchard, Leandro, et al. (2021) on the other hand, use a probabilistic approach to assess the sustainability of the debt. Due to the complex nature of the development of debt ratios, it no longer suffices to focus solely on absolute debt sustainability according to Blanchard, Leandro, et al. (2021). In their analysis, the focus is on the likelihood of escalating debt rather than the feasibility of stabilising debt.

Research on FRF suggests that countries characterised by a significant positive debt coefficient can be considered sustainable. However, whilst this is indicative of the current position, it is more important to look at future behaviour. To determine whether sustainability can be maintained, one should look at the *fiscal space*, which is defined as "room in a government's budget that allows it to provide resources for a desired purpose without jeopardizing the sustainability of its financial position or the stability of the economy" (Heller, 2005, p. 32). When this concept is related to the FRF, it can be seen as the distance between the current debt level and the debt limit for it to remain sustainable. Related to this is the concept of *fiscal fatigue*, which occurs when a country cannot longer use increased primary balances to counteract rising debt. Ghosh, Kim, et al. (2013) analysed data from 23 developed countries, to determine the debt limits. They found limits ranging from 150% to 250%, with large differences in fiscal space between countries. For the Netherlands, they found a debt limit of 190% of the *gross domestic product* (GDP). These findings are supported by Fournier and Fall (2015), who even reported debt limits above 200% of GDP for the Netherlands. The main driver of differences in debt limits was found to be the difference between the interest rate and the growth rates (Ghosh, Kim, et al., 2013). Furthermore, their findings suggest that the debt limit is predominantly dependent on country-specific fiscal behaviour. However, it should be noted that there is no consensus on the existence of fiscal fatigue; other research has indicated increased fiscal responsiveness in response to elevated debt levels accrued during the financial crisis (Baldi and Staehr, 2016; Checherita-Westphal and Žd'árek, 2017).

More recently, the *debt sustainability analysis* (DSA) has become popular, which is another quantitative framework used to assess a country's ability to manage its debt obligations. DSA focusses on the future development of fiscal stability and economic growth. Debt sustainability analyses allow for uncertainties to be incorporated into future projections of debt ratios. Various policy institutions have conducted a DSA to obtain the distribution of debt developments under current fiscal policy. Bouabdallah et al. (2017) performed a stochastic and deterministic DSA for euro area souvereigns and created a heat map that indicates the risks of individual countries on behalf of the ECB. In addition to the baseline model, the ECB also incorporated FRF into the DSA framework. Using their estimated FRF, Berti et al. (2016) conducted a debt sustainability analysis with integrated country-specific FRF to create alternative projections. Compared to the no-policy change scenario, incorporating FRF showed on average an increase in projected debt ratios. In addition, IMF (2022) has created a DSA framework to assess debt sustainability, detect vulnerability, and if necessary analyse alternative policy trajectories. Similar research has been conducted by the European Commission, which emphasised the importance of the DSA framework, concluding that the DSA framework should become the backbone of EU fiscal rules (Heimberger, 2023).

In addition, in the current debate on the definition of sustainable debt, there is controversy about the implications of current interest rates on debt that fall short of growth rates ($r < g$). Historically, it was assumed that (on average) $r > g$, in analysing the sustainability of the debt. If $r < g$, interest payments on debt outpace economic growth, which highlights the importance of prudent fiscal and monetary policy. However, recently, most developed economies have been characterised by rates for which $r < g$. In this case, debt sustainability appears to be less of an issue, since the debt ratio will always reach a steady state as long as the primary deficit does not continue to rise and r remains below g (see Willems and Zettelmeyer (2022) for an elaborate explanation). According to Blanchard (2019), very low interest rates reduce the cost of public debt, which means that higher debt levels are no longer necessarily associated with higher fiscal expenses. However, this has caused controversy among economists. First, higher debt levels were claimed to come with a higher risk of rollover (more debt must be refinanced), even when $r < g$ (Mauro and Zhou, 2020; Moreno Badia et al., 2020). Second, an increase in debt levels may cause a crowding-out effect (resulting from a rising interest rate caused by higher debt levels). Additionally, even if debt sustainability is no longer a problem if the interest rate is lower than the growth rate, it is still uncertain whether this will remain. Therefore, to determine the optimal fiscal policy, both scenarios must be taken into account.

2.3 Optimal fiscal policy

The academic field on optimal fiscal policy consists of theoretical and empirical aspects, which will be discussed in the following section. While theoretical models primarily focus on optimising a restrictive objective function, often centred on the optimal redistribution of resources, they may not fully account for empirical considerations, such as the influence of the business cycle and the presence of fiscal multipliers. As such, besides determining optimal fiscal policy from a theoretical perspective, this section will also look at (optimal) fiscal policy in an empirical framework, taking into account societal preferences and practical restrictions to complement the theoretical insights.

2.3.1 Theoretical optimisation

Extensive research has been carried out on determining welfare-maximising fiscal policy rules using (adaptations of) Ramsey allocation planners. The Ramsey model is an economic framework used to study the optimal allocation of resources over time to maximise individual welfare or utility using representative agents (Ramsey, 1928). Starting with Barro (1979), who considered how different fiscal policies, including public debt, affect the welfare of current and future generations. His simplified model examines the interactions between different generations of individuals in an economy. His key finding highlights that maintaining consistent taxes over time is preferred to fluctuating taxes, due to convex costs of taxes. According to him, this policy can lead to more predictable and stable welfare outcomes for all generations. Hence, in the presence of convex tax costs, Barro (1979) suggests that it is preferable to absorb shocks through fluctuations in government debt, rather than maintaining volatile tax schemes. This implies that debt follows a random walk.

While this model has been highly influential and valuable, it relies on some very strong assumptions, e.g. perfect foresight and rational expectations, homogeneous households, and behavioural aspects Lucas Jr and Stokey (1983) extended on Barro's model by incorporating infinite horizons, continuous time, money, and policy rules. Their research has offered valuable insights on the interplay of fiscal and monetary policies, government debt dynamics, and intergenerational welfare, yielding three key lessons: 1) the government's present value budget constraint must be satisfied (i.e., budget deficits may occur in specific periods, but they must be offset with surpluses in other periods); 2) continuous budget balance is not imperative; 3) state-contingent debt is vital for optimal fiscal policy in complete markets (Lucas Jr and Stokey, 1983, p. 77).

In contrast to the aforementioned research with complete markets, Aiyagari et al. (2002) assumed incomplete markets with one-period risk-free government borrowing, where agents face uncertainty and cannot fully protect themselves against idiosyncratic risk. Based on this model, they confirmed that the first and second lessons drawn by Lucas Jr and Stokey (1983) roll over in their adapted model, while they modify the importance of state-contingent debt depending on the properties of the incomplete market model. The most recent research related to this strand is conducted by Bhandari et al. (2017) who analysed an economic framework that accounts for stochastic interest rates, revenue and growth. Their model is largely in line with Aiyagari et al. (2002), however, Bhandari et al. (2017) introduced a model in which agents cannot bear the financial burden of paying positive lump sum taxes. The results of their optimisation process indicate that the most favourable approach is to maintain a debt target, which is gradually attained in the long run. This implies that the government should take a slow and cautious approach when aiming at its desired debt level.

Hence, it can be concluded from a theoretical perspective that optimal fiscal policy should lead to a consistent (neutral or explicit) debt target in the long run. These models, however, do not take into account empirical and practical constraints and the existence of business cycles.

2.3.2 Empirical considerations

Empirical research on optimal fiscal policy has a strong connection to the relationship between fiscal policy and economic fluctuations. From a macroeconomic point of view, fiscal and monetary policies are two essential tools that governments and central banks use to manage and stabilise the economy of a country and mitigate economic shocks (Blanchard, Amighini, et al., 2017). Fiscal stimulus, i.e., increased government spending and/or reduced taxes, is meant to increase disposable income, which should cause aggregate demand to rise. On the contrary, fiscal consolidation allows the government to reduce economic activity, which can help control inflation. In addition to fiscal policy, the economy can also be regulated using monetary policy, which involves controlling the money supply and interest rates (Blanchard, Amighini, et al., 2017). Monetary policy aims to influence the cost and availability of money in the economy to control inflation, economic growth, and unemployment. In contrast to fiscal policy, monetary policy is set by the central bank.

Empirically, it was found that the implementation and design of these policies come with many restrictions for central banks and governments. First, the European Central Bank (ECB) sets the monetary policy for all euro nations, which implies that all countries in the monetary union share the same monetary policy. Due to this structure, the adaptive power of monetary policy is limited (Hartmann and Smets, 2018). Additionally, a *zero lower bound* was imposed on the interest rate, which restricts the ECB to set a nominal interest rate below zero percent. This was found problematic during the financial and euro crisis, when central banks could no longer spur economic growth by reducing the interest rate, a concept also known as the *liquidity trap* (DeLong and Tyson, 2013).

Furthermore, in order to determine which policy to impose to stabilise the economy, policy makers have to deal with a large degree of uncertainty. First, when making policy decisions, policy makers must rely on educated estimates of the development of macroeconomic variables that affect the economy, such as growth, inflation, and interest. The academic domain pertaining to growth models alone is of substantial magnitude (see Cerra et al. (2023) for an extensive overview of growth models and business cycles). Furthermore, recent events have shown that the uncertainty embedded in inflation can also take on large magnitudes (CBS Statistics Netherlands, 2023). Akin predicting growth, large advances are made in predicting inflation (Stock and Watson, 1999; Koop and Korobilis, 2012). However, it was found that the creation of accurate forecasts remains challenging, even within short time horizons (Cecchetti, 1995; Cecchetti et al., 2000).

Moreover, there is still an ongoing debate about the effects of macroeconomic interventions. This effect is commonly referred to as the fiscal multiplier of the intervention. Fiscal multipliers measure the (short-term) effect of government spending on the economy measured by GDP (Batini et al., 2014). It indicates the percentage changes as a result of a 1% change in government spending as a percentage of GDP. A larger multiplier implies a larger impact on the output. Previous research has demonstrated that the effectiveness of fiscal policy is not unambiguous (Brainard, 1967; Ilzetzki et al., 2013). More recent literature suggests that fiscal multipliers are asymmetric and cyclical (e.g. Auerbach and Gorodnichenko (2012), Barnichon et al. (2022), Berge et al. (2021), Cacciatore et al. (2021), and Fotiou (2022)). Katz and Bettendorf (2023) conducted an extensive literature review of empirical analyses, including (panel) analyses of VS, EU, and OECD, investigating the impact of government spending. The results of their study showed that multipliers tend to be higher during economic downturns and when linked to restrictive policy modifications, due to the presence of inflexible wages and borrowing limitations for households. Combining these findings, during recessions, restrictive policy multipliers are estimated between 1 and 2.7, and expansionary policy multipliers between 0.5 and 4.5. For non-recession periods, restrictive policy estimates range from 1 to 2.7, and expansionary policy estimates from -0.3 to 2.2.

Variations in the fiscal multiplier can be attributed to the fact that the effect depends greatly on the specification of the model and the underlying assumptions, leading to diverse outcomes among various models (Blanchard, Amighini, et al., 2017). Even in the absence of uncertainty in all other variables, it remains challenging to distinguish the effect of interventions, as policy responses involve behavioural aspects that complicate the analysis. Given the presence of anticipating agents (e.g. households and firms), identifying suitable policy measures should be considered as a strategic game. In this framework, the problem can be seen as a (repeated) game with asymmetric information and agents with different (political) incentives. The results of such analyses stress the importance of credibility and commitment of the government to ensure their policy success (Backus and Driffill, 1985; Braun and Tommasi, 2004; Saulo et al., 2013).

2.4 Integration of the literature

Over time many different comprehensive models have been developed, which have provided us with valuable information regarding the current financial position, debt sustainability, and optimal fiscal policy. There seems, however, to be a sharp division between these strands. Academic research on FRF has greatly contributed to the understanding of (historical) fiscal policy behaviour and the underlying dynamics of macroeconomic variables. However, this research mainly focusses on past actions rather than projecting future scenarios. In addition, literature on debt sustainability has given us valuable tools to evaluate current policy, such as the DSA framework and the recent probablistic approach towards debt sustainability. Lastly, the academic field surrounding the optimisation of fiscal policy has provided us with the insight that theoretically a consistent debt target should be aimed at, however, this does not account for business cycles or the presence of fiscal multipliers.

This thesis attempts to incorporate these different strands, looking for optimal fiscal policy not only from an individual welfare point of view, incorporating the empirical considerations, and taking into account the sustainability requirements. It will build on existing literature using the DSA framework to compare different fiscal policies, based on theoretical optimisation outcomes, and identify the most appropriate policy given the economic conditions. The understanding acquired from welfare optimisation models and empirical fiscal response functions will be used to simulate anchors for fiscal policy.

3 Theoretical framework

Firstly, this section will provide an overview of the theoretical framework employed for forecasting debt ratios and other macroeconomic variables that influence debt dynamics. Consequently, there will be a detailed description of the objective of this analysis and the benchmark scenarios used to compare the optimised fiscal policy. Lastly, there will be a discussion on the optimisation approach.

3.1 Debt sustainability analysis framework

Determining reliable estimates of the NPV of government assets and its future usability pose a great challenge for evaluating the sustainability of debt (Bouabdallah et al., 2017). As such, analytical research often relies on long-term projections of debt ratios to create expectations of the future capacity to service debt. A commonly used debt accumulation equation is given as follows:

$$D_t = (1 + r_t)\, D_{t-1} - PB_t, \qquad \forall t = 1, ..., T. \tag{1}$$

This equation calculates the current level of government debt (D_t) by multiplying the debt stock in the previous period (D_{t-1}) by the implicit average interest rate (r_t) and subtracting the primary balance (PB_t), which is government revenue minus spending (excluding interest payments) in the year t. The government debt consists of all outstanding obligations of the (local) government and social institutions (Rijksoverheid, n.d.). In addition, the implicit interest rate is a weighted average of the interest rates on the debt acquired previously. Governments can acquire debt by selling interest-bearing government bonds. The interest rate on the government differs over time, depending on market conditions, policy considerations, and the term structure of the bond. Hence, the total interest rate on the debt stock is not characterised by a single rate; as such, it is common to use the weighted average of the interest payments as interest rate.

As debt is commonly analysed as a percentage of GDP, all terms are divided by nominal GDP (Y_t) – a compound measure of the size of the economy – which results in

$$\frac{D_t}{Y_t} = (1 + r_t)\frac{D_{t-1}}{Y_t} - \frac{PB_t}{Y_t}, \qquad \forall t = 1, ..., T, \tag{2}$$

$$\frac{D_t}{Y_t} = (1 + r_t)\frac{D_{t-1}}{(1 + g_t)\, Y_{t-1}} - \frac{PB_t}{Y_t}, \qquad \forall t = 1, ..., T, \tag{3}$$

$$d_t = \frac{1 + r_t}{1 + g_t} d_{t-1} - pb_t, \qquad \forall t = 1, ..., T. \tag{4}$$

To obtain (3) from (2), Y_t is replaced by $(1 + g_t)Y_{t-1}$ in the first term of the right side of the equation, which can be explained by the fact that GDP in time t (Y_t) can be calculated by multiplying GDP in the previous period (Y_{t-1}) by the nominal growth factor $(1 + g_t)$. Furthermore, (4) can be obtained using lowercase symbols to indicate variables expressed as percentage of GDP and rearranging the terms.

The model employs nominal terms for all variables, which means that the variables are not corrected for inflation. Adopting nominal terms makes the projection more straightforward and allows for a better understanding of the debt dynamics. In addition, international budgeting rules and financial obligations are commonly in nominal terms, which also advocates for an analysis in nominal terms. The downside of nominal terms is the lack of correlation between growth and interest rates. In the real world, high growth rates are often accompanied by high interest rates and vice versa due to the fact that inflation affects both variables. Without taking inflation into account, it may occur that circumstances featuring very high interest rates and low growth rates arise, creating overpessimistic scenarios. This should be taken into account when using this model.

In order to project debt, the underlying macro-economic variables, (g_t, r_t and pb_t) must also be projected. Understanding the behaviour and interactions of these variables is of utmost importance for reliable projections.

3.1.1 Growth

Whereas traditionally growth theories relied on exogenous factors, growth is commonly seen as endogenous, which implies that long-term growth of economies depends on internal factors (Blanchard, Amighini, et al., 2017). The introduction of endogenous growth theory has spurred a whole new class of research (see, e.g., Howitt (2010), Romer (1994), and Shaw (1992)). Although there is still an ongoing debate on exploring and refining the determinants and interactions of growth, endogenous growth theory has been found to be very influential in shaping policy discussions and understanding the role of institutions. A consensus was reached about the importance of internal factors in modelling economic growth. As such, in this model, growth rates will be simulated endogenously using the following equation

$$g_t = c_1 g_{t-1} + (1 - c_1) g_t^p + c_2 \Delta DP_t - c_3 OG_{t-1} + \omega_t, \qquad \forall t = 1, ..., T. \tag{5}$$

Within this model, growth (g_t) is projected as a weighted average of the growth lag (g_{t-1}) and potential GDP growth (g_t^p). Additionally, the impact of changes in fiscal policy (ΔDP_t) is also included, using a fiscal multiplier denoted by c_2. To guarantee a closed output gap, a cyclical closure is added. The output gap is defined as the difference between actual and potential output as a percentage of potential output ($OG_t = (Y_t - Y_t^p)/Y_t^p, \forall t = 1, ..., T$). The potential output level refers to the maximum sustainable level of economic output that an economy can achieve when all resources, including labour and capital, are fully utilised (Jahan and Mahmud, 2013). The actualised output Y_t tends to float around the potential output level Y_t^p, which will be denoted with a superscript p. In contrast to real output, potential output cannot be observed and must be determined based on the (country-specific) production function. A common approach is to compare potential employment with structural employment adjusted for equilibrium unemployment to obtain potential output (Dicou, 2022). The output gap can be either positive (actual output surpasses potential output) or negative (vice versa).

The growth model employed is in line with the endogenous growth model specified by the ECB in their DSA model. All coefficients are set in accordance with estimates of Bouabdallah et al. (2017). The coefficient c_1 represents the autoregressive coefficient of g and is set at 0.55, c_2 represents the fiscal multiplier, set at -0.55. Lastly, c_3 is the elasticity with respect to the output gap, set at 0.4. In addition to the endogenous growth model, a shock (ω_t), is also introduced, which is drawn from the historical distribution of shocks (centred to zero).

3.1.2 Interest payments

The interest payment on the currently outstanding government debt is determined by the interest rate r_t. This rate is a weighted average of the historical interest rates as the interest rate of debt is determined at the moment of acquisition. Therefore, the interest payment is not equal to the current market interest rate. Given that the current average interest rate is a weighted average of historical interest rates, it must be autocorrelated. This was also empirically confirmed using data from the *CPB Netherlands Bureau for Economic Policy Analysis* from 1981 to 2022. The autocorrelation of the first lag of the historical average interest payment was 0.933. Furthermore, the *Augmented Dicky Fuller* test failed to reject the presence of a unit root ($ADF = -0.196, p = 0.94$). Therefore, the average interest rate on debt is simulated using a random walk with *zero lower bound*, which is given by

$$r_t = \max\{0, r_{t-1} + i_t\}, \qquad \forall t = 1, ..., T. \tag{6}$$

Updates of the average interest rates (i_t) are drawn randomly from the historical distribution of innovations with an expected value of zero. In addition, the *zero lower bound* is induced by the maximum function, which prevents negative interest rates.

3.1.3 Primary balance

In contrast to growth and interest, the primary balance can be influenced by external parties, such as policymakers. Primary balance can be broken down into two parts, structural primary balance and cyclical components. First, the cyclical components refer to changes in government revenue and expenditures that occur as a result of natural fluctuations in the economy's business cycle. Commonly, in periods of economic expansion, revenue tends to increase and expenditure decrease, while in periods of contractions the opposite occurs. Cyclical components are often the automatic stabilisers of the system - mechanisms that adjust government revenue and spending in response to changes in the economy to dampen impact of fluctuations (Mohl et al., 2019). Second, the structural component of the primary balance refers to the part of the primary balance that is not affected by short-term economic fluctuations or cyclical changes. This component looks at the sustainable level of government spending and revenue over time (Abdel-Kader, 2013).

In this analysis, the primary balance (pb_t) will be modelled using

$$pb_t = DP_t + \pi_t + \beta\omega_t + \epsilon_t, \qquad \forall t = 1, ..., T, \tag{7}$$

in which DP_t refers to the discretionary policy imposed to implement explicit policy changes with respect to the basic path π_t. Discretionary policy can be used to respond to changes in the environment and to implement explicit policy decisions. The basic path represents the expected long-term primary balance from time t onwards. The presence of a basic path allows us to incorporate financial trends, such as ageing costs. Lastly, the primary balance is also characterised by uncertainty, consisting of two parts: $\beta\omega_t + \epsilon_t$. Primary balance is affected by (unexpected) growth shocks (ω_t) with elasticity β. Elasticity was found to be 0.6, based on historical data from the Netherlands, and confirmed by the European Commission (Mourre et al., 2014). Furthermore, ϵ_t is the shock in the primary balance independent of growth, drawn from the historical distribution of shocks with the first moment equal to zero.

3.2 Benchmark Scenarios

In order to evaluate the optimised fiscal policy, four benchmark scenarios will also be simulated. Benchmark scenarios show how the debt ratio evolves under different (academically determined) policy rules.

3.2.1 Baseline scenario

The baseline scenario shows how the debt ratio would evolve according to the model described above without explicit government interventions or constraints, i.e., $DP_t = 0, \forall t = 1, ..., T$. Thus, the primary balance is solely determined by the basic path and (economic) shocks.

3.2.2 Stability and Growth Pact Scenarios

The Stability and Growth Pact (SGP) is an agreement among member states of the European Union to ensure fiscal discipline and sustainable government finances. According to the SGP, each member state in the monetary union must adhere to two budgetary rules with respect to its deficit and debt. The debt ratio must remain below 60% of GDP, while the deficit cannot exceed 3% of GDP (including interest payment). Violation of these rules can lead to (monetary) sanctions (Blanchard, Amighini, et al., 2017; European Commission, 2023). However, strict enforcement of these static restrictions was found to be difficult. Along the way, various reforms were implemented, incorporating several exceptions to create more fiscal space for member states to adapt to changing economic circumstances (European Commission, 2023).

Although the static budgetary rules of the SGP were never strictly invoked, it remains interesting to depict how debt and primary balances would evolve if a country strictly adheres to the budgetary rules. To simulate what would happen if member states strictly adhere to these rules, ad hoc constraints are imposed in the model. This implies that as soon as a constraint is violated, corrective action is immediately taken.

The ad hoc debt ratio constraint can be implemented using

$$DP_t = \max\{d_{t-1} - UB, 0\}, \qquad \forall t = 1, ..., T. \tag{8}$$

This equation evaluates the debt ratio at the end of the previous period. If the constraint of the debt ratio is violated, then the adjustment term (DP_t) is invoked, modifying the primary balance in this period in such manner that the excess debt in the previous period is compensated. For the SGP debt constraint, an upper limit of 60% is imposed ($UB = 0.60$).

For the deficit constraint, it must be assessed whether the current primary deficit (including interest payments) remains below the constraint $\underline{PB}$. Interest payment can be obtained by multiplying the current implicit interest rate by the debt stock at the end of the previous period. Whereas the debt ratio can be addressed by implementing a form of backward compensation, the deficit constraint must be rectified in the present period, which complicates the procedure. Given that it is impossible to have all information in the current period to calculate the realised primary deficit and interest payment, this analysis will resort to the following proxy:

$$DP_t = \max\{\underline{PB} - (pb_t - \hat{R}_t), 0\} \tag{9}$$

$$= \max\{\underline{PB} - \pi_t - \beta\omega_t - \epsilon_t + \hat{R}_t, 0\}, \qquad \forall t = 1, ..., T. \tag{10}$$

This equation is based on the assumption that the growth shock (ω_t) and the primary balance shock (ϵ_t) are known before the end of the year and that the government can take corrective action in the same year. Moreover, as interest payments as a percentage of GDP ($R_t = \frac{r_t D_{t-1}}{Y_t}, \forall t = 1, ..., T$) are not yet known, expected interest payments $\hat{R}_t = \frac{\hat{r}_t D_{t-1}}{\hat{Y}_t}, \forall t = 1, ..., T$ will be used, with $\hat{r}_t = r_{t-1}, \forall t = 1, ..., T$, which is the expected interest rate conditional on $t-1$. Furthermore, the expected value of GDP is obtained by multiplying the GDP in the previous period by the potential growth rate ($\hat{Y}_t = (1 + g_t^p) Y_{t-1}, \forall t = 1, ..., T$). Using this proxy, the primary deficit constraint is enforced by invoking the adjustment term (DP_t) when the primary balance minus the interest costs exceeds the deficit constraint. In addition, the default notation will be *primary balance* instead of *primary deficit*; the primary deficit can be obtained by taking the negative counterpart of the primary balance. Similarly to the debt-ratio constraint, the primary balance will be adjusted in the event of violation of the constraint.

3.2.3 Neutral debt target

The neutral debt target scenario is based on the academic literature indicating that in the long term, the government should aim for a consistent debt ratio (Barro, 1979). A constant debt ratio can be achieved by adjusting the primary balance in such a way that, in expectation, the debt ratio in the upcoming period matches the debt ratio in the present period, conditional on the previous period ($\mathcal{F}_{t-1}$). This leads to the following relation:

$$\mathbb{E}\left[d_t | \mathcal{F}_{t-1}\right] = d_{t-1}, \qquad \forall t = 1, ..., T. \tag{11}$$

The left side of this equation can be rewritten into

$$\begin{aligned}
\mathbb{E}\left[d_t | \mathcal{F}_{t-1}\right] &= \mathbb{E}\left[\frac{1+r_t}{1+g_t} d_{t-1} - pb_t \middle| \mathcal{F}_{t-1}\right] \\
&= \mathbb{E}\left[\frac{1+r_t}{1+g_t} d_{t-1} \middle| \mathcal{F}_{t-1}\right] - \mathbb{E}\left[pb_t | \mathcal{F}_{t-1}\right] \\
&= \mathbb{E}\left[\frac{1+r_t}{1+g_t} \middle| \mathcal{F}_{t-1}\right] d_{t-1} - \mathbb{E}\left[pb_t | \mathcal{F}_{t-1}\right].
\end{aligned}$$

Rearranging the terms indicates that the expected primary balance should be equal to

$$\mathbb{E}\left[pb_t | \mathcal{F}_{t-1}\right] = \left[\mathbb{E}\left(\frac{1+r_t}{1+g_t}\right) - 1 \middle| \mathcal{F}_{t-1}\right] d_{t-1}. \tag{12}$$

Using (7) to obtain the expected value of primary balance in the model, we obtain

$$\mathbb{E}\left[pb_t | \mathcal{F}_{t-1}\right] = \mathbb{E}\left[DP_t + \pi_t + \beta\omega_t + \epsilon_t | \mathcal{F}_{t-1}\right] \tag{13}$$

$$= \mathbb{E}\left[DP_t | \mathcal{F}_{t-1}\right] + \mathbb{E}\left[\pi_t | \mathcal{F}_{t-1}\right] \tag{14}$$

$$= DP_t + \pi_t. \tag{15}$$

This can be explained by the fact that the (conditional) expected value of ω_t and ϵ_t is equal to zero for all t. In addition, DP_t and π_t are independent of time $t-1$ and are not influenced by any stochasticity.

Hence, to fulfil the condition that the primary balance is consistent in expectation, it must be ensured that

$$DP_t + \pi_t = \left[\mathbb{E}\left(\frac{1+r_t}{1+g_t}\right) - 1 \middle| \mathcal{F}_{t-1}\right] d_{t-1}, \tag{16}$$

$$DP_t = \left[\mathbb{E}\left(\frac{1+r_t}{1+g_t}\right) - 1 \middle| \mathcal{F}_{t-1}\right] d_{t-1} - \pi_t. \tag{17}$$

However, this requires the expected value of $\frac{1+r_t}{1+g_t}$, conditional on $\mathcal{F}_{t-1}$. Obtaining the expected value of the ratio is computationally intense and challenging due to the interaction between primary balance and growth embedded in the model. As such, the model employed in this thesis will use a simplification to approximate the expected value, using

$$DP_t = \left(\frac{1 + \hat{r}_t}{1 + \hat{g}_t} - 1 \right) d_{t-1} - \pi_t, \qquad \forall t = 1, ..., T, \tag{18}$$

in which $\hat{r}_t$ is the expected value of r_t conditional on $t-1$, which is equal to r_{t-1}, and $\hat{g}_t$ is the long-term expected value of growth, proxied by the expected long-term value of growth (g_t^p). Although this is a strong assumption in the model, an ex post evaluation of this shortcut indicated that the difference is relatively small, see Appendix A. In addition, π_t is subtracted to determine the adjustment relative to the basic path. It should be noted that the term $\left(\frac{1+\hat{r}_t}{1+\hat{g}_t} - 1 \right) d_{t-1} = \left(\frac{1+\hat{r}_t}{1+\hat{g}_t} \right) d_{t-1} - d_{t-1}$ can be interpreted as the expected change in the debt ratio due to the interest and growth dynamic, since the first term ($\left(\frac{1+\hat{r}_t}{1+\hat{g}_t} \right) d_{t-1}$) is the expected new debt ratio from which the old debt ratio (d_{t-1}) is subtracted.

3.2.4 Explicit debt target

The explicit debt target is similar to the neutral debt target, except in this scenario the debt should converge to a predetermined debt target rather than remaining consistent in the long term. In this scenario, the debt target is set at 60% of GDP, which is in line with the SGP budgetary rule. Furthermore, according to the excessive debt procedure, if the restriction is violated, this excess must be reduced by 1/20 on average every year. This leads to the following function for the adjustment of primary balance

$$DP_t = \left(\frac{1 + \hat{r}_t}{1 + \hat{g}_t} - 1 \right) d_{t-1} + \xi_t - \pi_t, \qquad \forall t = 1, ..., T, \tag{19}$$

$$\xi_t = (d_{t-1} - 0.60)/20, \qquad \forall t = 1, ..., T. \tag{20}$$

3.3 Objective

Using stochastic optimisation, the objective of this thesis is to determine fiscal policy that satisfies multiple objectives. First, fiscal policy aims to create macroeconomic stability, which requires consistent fiscal policy (i.e., no sudden contractions nor expansions). Inconsistent budget allocations result in suboptimal policy outcomes. When policy decisions oscillate greatly over time, their efficacy is frequently compromised. Hence, fiscal policy and especially discretionary policy – deliberate and active decisions made by policymakers to adjust government spending and taxation in response to short-term economic conditions – should promote stabilisation (Studiegroep Begrotingsruimte, 2020).

In addition, the aim is to avoid situations in which debt becomes uncontrollable and grows rapidly. Minimising default risk is essential to maintain fiscal sustainability and macroeconomic stability. High debt ratios are detrimental to the financial position of a country and reduce the fiscal space that countries have to mitigate economic shocks (Studiegroep Begrotingsruimte, 2020). However, low debt ratios are not always desired either, given that low debt ratios require fiscal consolidation, which has a negative effect on economic growth (Jacobs, 2015; Teulings, 2016; Teulings, 2020b). Moreover, given the current low interest rates, it may be beneficial to borrow more money to support macroeconomic trends (Caballero et al., 2017).

Lastly, ideally, fiscal policy is countercyclical. Countercyclical policy diminishes the business cycle and can dampen the consequences of economic shocks, especially in the presence of (large) positive fiscal multipliers. Due to government interventions in through periods, unemployment can be diminished which stimulates the economy and diminishes the impact of negative shocks. According to Katz and Bettendorf (2023), the multiplier effect is especially high during periods, which makes countercyclical interventions even more valuable. Furthermore, recently Cerra et al. (2023) found extensive evidence for the presence of hysteresis in their review of the current theoretical and empirical literature. Hysteresis refers to the idea that temporary shocks or disruptions can have long-lasting effects on its growth potential of an economy. This suggests that the impact of negative economic shocks can persist even after the initial shock has subsided, leading to a reduction in the economy's long-term growth rate. In contrast to the convention that the wrong intervention would only induce increased GDP volatility, the presence of hysteresis would imply that errors could have permanent effects. Hence, it can be concluded that in this new framework the need for countercyclical policy should be further emphasised.

3.3.1 Discretionary Policy function

This analysis will employ a linear function form for the discretionary policy component (DP_t), which will be used to identify fiscal policy that meets the desired objectives. In the welfare optimising literature, the findings of Barro (1979) were very influential, advocating the preservation of a constant debt ratio. Subsequent research Bhandari et al. (2017) concluded that an explicit long-term debt target should be aimed at. Moreover, as the aim is to construct a countercyclical policy, the discretionary primary balance function must also contain a term reflecting the current state of the economy. This has led to the following functional form for the adjustment term

$$DP(\boldsymbol{\theta})_t = \theta_0 d_{t-1} + \theta_1 OG_{t-1} + \theta_2 \left(\frac{1+\hat{r}_t}{1+\hat{g}_t} - 1 \right) d_{t-1} + \theta_3 (d_{t-1} - 0.60) - \pi_t, \forall t = 1, ..., T. \tag{21}$$

In this equation, the vector $\boldsymbol{\theta} = [\theta_0, \theta_1, \theta_2, \theta_3]$ represents the relative weights of the different components affecting the adjustment (DP_t) of primary balance. The adjustment term can be seen as an adaptation of Barro (1979) (which can be achieved with $\theta_2 = 1$ and $\theta_0 = \theta_1 = \theta_3 = 0$) combined with an implementation of the findings of Bhandari et al. (2017) advocating a long-term debt target (set at 60% in this model). Moreover, the lagged output gap (OG_{t-1}) is included as a measure of the state of the economy. As such, discretionary policy will be a function of the debt ratio at the end of the previous period, the lagged output gap, an interaction term between r, g and d_{t-1} which is constructed in line with the debt target scenarios, and a small adjustment term to introduce an explicit debt target.

3.3.2 Objective function

This functional form of discretionary policy combined with the various objectives has resulted in the following problem specification. Firstly, the aim is to minimise changes in primary balance to reduce sudden contractions and expansions. This can be achieved by minimising the standard deviation of the first difference of the policy adjustments ($\sigma_{\Delta DP(\boldsymbol{\theta})}$). Changes in discretionary policy ($\Delta DP(\boldsymbol{\theta})_t = DP(\boldsymbol{\theta})_t - DP(\boldsymbol{\theta})_{t-1}$) indicate additional expansions ($\Delta DP(\boldsymbol{\theta})_t > 0$) or contractions ($\Delta DP(\boldsymbol{\theta})_t < 0$), while a constant adjustment term means that there are no additional policy changes with respect to the previous period. As the aim is to minimise the volatility in changes in fiscal policy, the objective of this thesis is given by:

$$\min_{\boldsymbol{\theta}\in\mathbb{R}^4} J(\boldsymbol{\theta}) = \min_{\boldsymbol{\theta}} \sigma_{\Delta DP(\boldsymbol{\theta})} \tag{22}$$

$$= \min_{\boldsymbol{\theta}} \sqrt{\frac{\sum_{t=2}^{T}\left(\Delta DP(\boldsymbol{\theta})_t - \overline{\Delta DP(\boldsymbol{\theta})}\right)^2}{T-2}}. \tag{23}$$

In this equation, $\overline{\Delta DP(\boldsymbol{\theta})}$ represents the average of the vector containing $\Delta DP(\boldsymbol{\theta})_t = DP(\boldsymbol{\theta})_t - DP(\boldsymbol{\theta})_{t-1}$ for $t = 2, ..., T$, obtained using

$$\overline{\Delta DP(\boldsymbol{\theta})} = \frac{1}{T-1}\sum_{t=2}^{T} \Delta DP(\boldsymbol{\theta})_t. \tag{24}$$

Note that ΔDP_t starts at $t = 2$, because of the difference term.

Furthermore, to ensure that debt levels do not explode, nor reach such low levels that it can be harmful to the economy, the debt ratio will be restricted: $d_t \in [30\%, 80\%]\, \forall t \in 1, ..., T$, which restricts the admissible path of the debt ratio. Debt constraints can be chosen depending on the risk preference of a country. In addition, the constraints can also be used to restrain debt ratios to a predetermined range (which is, for example, required for the Stability and Growth Pact). For this study, a loose lower limit was selected to provide the optimisation process with more space to locate an optimal solution. On the other hand, the upper bound is characterised by a tighter constraint to reduce the default risk. Moreover, from a political perspective, there is little support for high debt ratios (Teulings, 2020a). However, many economists agree that higher debt ratios can actually be beneficial (Bezemer, 2020). As such, it was chosen to allow the debt ratios to rise above 60% of the Stability and Growth Pact, but must remain below 80%.

Lastly, the degree of countercyclicality will not be implemented explicitly in the objective, but rather be used as an evaluation measure. Although fiscal policy is preferred to be countercyclical, it is difficult to determine an optimal level of countercyclicality. Additionally, given that fiscal policy must be set in advance and the timing of business cycles is hard to predict, it was chosen not to use this as the main objective.

3.4 Stochastic optimization

In order to find an optimum value for a stochastic problem, several stochastic optimisation techniques can be employed. Stochastic optimisation is a category of optimisation methods that leverage random sampling to estimate the objective function or gradients. Among the stochastic optimisation techniques, stochastic approximation (SA) is a well-known method. SA is an iterative optimisation technique used to approximate the solution of an optimisation problem when the objective function or its gradients are not deterministic (Ketkar, 2017). Given the random nature embedded in stochastic problems, the objective function or gradients cannot be computed exactly. As such, SA uses estimates of a random sample to approximate these values in each iteration.

Given an initial configuration $\boldsymbol{\theta}_0$, SA algorithms iteratively update estimates of the solution of the optimisation problem using

$$\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k + \epsilon_k G(\boldsymbol{\theta}_k), \tag{25}$$

until it has converged or a stopping criteria has been reached. In this equation, the solution $\boldsymbol{\theta}_k$ is iteratively updated with ϵ_k the stepsize - also known as the learning rate - and $G(\cdot)$ the descent direction. The descent direction of the algorithm can be any vector $d(\boldsymbol{\theta})$ such that $\nabla J(\boldsymbol{\theta})d(\boldsymbol{\theta}) < 0$. A commonly used driver for updates is the negative gradient $-\nabla J(\boldsymbol{\theta})$, for which $\nabla J(\boldsymbol{\theta})d(\boldsymbol{\theta}) < 0$ clearly holds. Furthermore, for the stepsize, the following criteria must hold:

$$\sum_{n=1}^{\infty} \epsilon_n = \infty \text{ and } \sum_{n=1}^{\infty} \epsilon_n^2 < \infty. \tag{26}$$

These criteria ensure that the series $\boldsymbol{\theta}_n$ is not bounded, implying that all possible values of $\boldsymbol{\theta}$ can be reached (Vázquez-Abad and Heidergott, 2023).

For stochastic problems, it is not possible to obtain the exact gradient. As such, stochastic gradient descent (SGD) algorithms use randomly sampled subsets of data to estimate the gradient of the objective function $J(\boldsymbol{\theta})$. For SGD, it holds that $G(\cdot)$ is determined by an estimate of the gradient $\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta})$.

3.4.1 FD estimator

There are several methods to estimate the gradient $\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta})$. A commonly used method is the central finite-difference approximation (FD), which evaluates the objective function at nearby points. FD computes estimates of the partial derivatives by taking the difference between the evaluations of the objective function at points slightly perturbed along each dimension and dividing it by the perturbation step size c_k, which results in

$$\nabla_{\boldsymbol{\theta_k}} J(\boldsymbol{\theta_k}) \approx \begin{bmatrix} \Delta_{\theta_{k,0}} J(\boldsymbol{\theta}_k) \\ \Delta_{\theta_{k,1}} J(\boldsymbol{\theta}_k) \\ \vdots \\ \Delta_{\theta_{k,n}} J(\boldsymbol{\theta}_k) \end{bmatrix}, \tag{27}$$

$$\Delta_{\theta_{k,i}} J(\boldsymbol{\theta}_k) = \frac{J(\theta_{k,0}, \theta_{k,1}, \ldots, \theta_{k,i} + c_k, \ldots, \theta_{k,n}) - J(\theta_{k,0}, \theta_{k,1}, \ldots, \theta_{k,i} - c_k, \ldots, \theta_{k,n})}{2c_k}, \tag{28}$$

in which $\Delta_{\theta_{k,i}} J((\theta)_k)$ represents the central finite difference estimate of the i-th partial derivative of the objective function, in iteration k of the gradient descent algorithm.

The FD estimator is relatively easy to implement, however, it is a biased estimator characterised by a considerable variance. This may reduce the efficiency of the optimisation process. The biased approximation is given by

$$\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k - \epsilon_k \left(\nabla_\theta J(\boldsymbol{\theta}_n)^T + \beta_n(\boldsymbol{\theta}_n) \right). \tag{29}$$

From (29) it can be observed that the descent direction consists of the negative gradient and a bias term ($\beta_n(\boldsymbol{\theta}_n)$. Biased estimators can be used in stochastic approximation algorithms as long as the bias term converges to 0 in the limit (i.e., $\beta_n(\theta_n) \to 0$) must be ensured) (Vázquez-Abad and Heidergott, 2023). This relates to a third condition on the stepsize and the bias:

$$\sum_{n=1}^{\infty} \epsilon_n \|\beta_n(\theta_n)\| < \infty. \tag{30}$$

This condition entails that the sum of products of ϵ_n and the norm of the bias term $\|\beta_n(\theta_n)\|$ to infinity is finite, ensuring that bias term converges. This is a necessary condition for an accurate and reliable gradient estimate. For the central FD estimator, the following holds (using Taylor series expansion)

$$J(\theta_{k,0}, \theta_{k,1}, \ldots, \theta_{k,i} \pm c_k, \ldots, \theta_{k,n}) = J(\boldsymbol{\theta_k}) \pm c_k \frac{\partial J(\boldsymbol{\theta}_k)}{\partial \theta_{k,i}} + \mathcal{O}(c_k^2). \tag{31}$$

Substituting this into (28), leads to

$$\Delta_{\theta_{k,i}} J(\boldsymbol{\theta}_k) = \frac{\left(J(\boldsymbol{\theta}_k) + c_k \frac{\partial J(\boldsymbol{\theta}_k)}{\partial \theta_{k,i}} + \mathcal{O}(c_k^2)\right) - \left(J(\boldsymbol{\theta}_k) - c_k \frac{\partial J(\boldsymbol{\theta}_k)}{\partial \theta_{k,i}} + \mathcal{O}(c_k^2)\right)}{2c_k} \tag{32}$$

$$= \frac{2c_k \frac{\partial J(\boldsymbol{\theta}_k)}{\partial \theta_{k,i}} + \mathcal{O}(c_k^2) - +\mathcal{O}(c_k^2)}{2c_k} \tag{33}$$

$$= \frac{\partial J(\boldsymbol{\theta}_k)}{\partial \theta_{k,i}} + \frac{\mathcal{O}(c_k^2)}{2c_k}. \tag{34}$$

Therefore, based on $\beta_k(\boldsymbol{\theta_k}) = \frac{\mathcal{O}(c_k^2)}{2c_k}$, it can be concluded that the bias term for each of the central finite difference estimators approaches zero, as the step size approaches zero. This makes the finite difference estimator a valid estimator for the gradient. As such, theoretically, the finite difference estimator should converge to the true (partial) derivative.

3.4.2 IPA estimator

Besides the FD estimator, the gradient can also be estimated using the *infinite perturbation analysis* (IPA) estimator. This estimator requires the objective function to be Lipschitz continuous, which is a necessary condition for the interchange of the derivative and expectation. Given the linear dependence in the problem, it can be concluded that Lipschitz continuity should be no problem. For simplicity, the derivation in this section makes use of the variance instead of the standard deviation; however, the same procedure can be applied to the standard deviation by inserting a square root and should result in a similar outcome.

Using the IPA estimator, the following partial derivatives can be obtained

$$\begin{aligned}
\frac{\partial}{\partial \theta_i} J(\boldsymbol{\theta}) &= \frac{\partial}{\partial \theta_i}\left[\frac{1}{T-2}\sum_{t=2}^{T}(\Delta DP_t - \overline{\Delta DP})^2\right] \\
&= \frac{1}{T-2}\sum_{t=2}^{T}\frac{\partial}{\partial \theta_i}(\Delta DP_t - \overline{\Delta DP})^2 \\
&= \frac{2}{T-2}\sum_{t=2}^{T}(\Delta DP_t - \overline{\Delta DP})\left(\frac{\partial}{\partial \theta_i}\Delta DP_t - \frac{\partial}{\partial \theta_i}\overline{\Delta DP}\right) \\
&= \frac{2}{T-2}\sum_{t=2}^{T}(\Delta DP_t - \overline{\Delta DP})\left(\frac{\partial}{\partial \theta_i}DP_t - \frac{\partial}{\partial \theta_i}DP_{t-1} - \frac{\partial}{\partial \theta_i}\overline{\Delta DP}\right), \quad \forall i = 0, 1, ..., 3,
\end{aligned} \tag{35}$$

which illustrates that the partial derivatives depend on the partial derivatives of $DP_t, \forall t = 1, ..., T$. Using the recursive relations between the variables, partial derivatives of discretionary policy term (DP_t) can be derived. Given that the initial values are constants, the following partial derivatives of $t = 0$ can be obtained

$$\frac{\partial}{\partial \theta_0} DP_0 = 0, \qquad \frac{\partial}{\partial \theta_1} DP_0 = 0, \qquad \frac{\partial}{\partial \theta_2} DP_0 = 0, \qquad \frac{\partial}{\partial \theta_3} DP_0 = 0.$$

Proceeding to the next period results in the following partial derivatives for DP_1

$$\begin{aligned}
\frac{\partial}{\partial\theta_0}DP_1 &= d_0 + \left(\theta_0 + \theta_2\left(\frac{1+\hat{r}_1}{1+\hat{g}_1} - 1\right) + \theta_3\right)\frac{\partial}{\partial\theta_0}d_0 + \theta_1\frac{\partial}{\partial\theta_0}OG_0 \\
&\quad + \frac{\partial}{\partial\theta_0}\max\{d_0 - UB, 0\} + \frac{\partial}{\partial\theta_0}\min\{d_0 - LB, 0\}, \\
\frac{\partial}{\partial\theta_1}DP_1 &= OG_0 + \left(\theta_0 + \theta_2\left(\frac{1+\hat{r}_1}{1+\hat{g}_1} - 1\right) + \theta_3\right)\frac{\partial}{\partial\theta_0}d_0 + \theta_1\frac{\partial}{\partial\theta_1}OG_0 \\
&\quad + \frac{\partial}{\partial\theta_1}\max\{d_0 - UB, 0\} + \frac{\partial}{\partial\theta_1}\min\{d_0 - LB, 0\}, \\
\frac{\partial}{\partial\theta_2}DP_1 &= \left(\frac{1+\hat{r}_1}{1+\hat{g}_1} - 1\right)d_0 + \left(\theta_0 + \theta_2\left(\frac{1+\hat{r}_1}{1+\hat{g}_1} - 1\right) + \theta_3\right)\frac{\partial}{\partial\theta_2}d_0 + \theta_1\frac{\partial}{\partial\theta_2}OG_0 \\
&\quad + \frac{\partial}{\partial\theta_2}\max\{d_0 - UB, 0\} + \frac{\partial}{\partial\theta_2}\min\{d_0 - LB, 0\}, \\
\frac{\partial}{\partial\theta_3}DP_1 &= (d_0 - 0.60) + \left(\theta_0 + \theta_2\left(\frac{1+\hat{r}_1}{1+\hat{g}_1} - 1\right) + \theta_3\right)\frac{\partial}{\partial\theta_3}d_0 + \theta_1\frac{\partial}{\partial\theta_3}OG_0 \\
&\quad + \frac{\partial}{\partial\theta_3}\max\{d_0 - UB, 0\} + \frac{\partial}{\partial\theta_3}\min\{d_0 - LB, 0\}.
\end{aligned}$$

These functions require partial derivatives of the auxiliary variables d_0, $OG_0, \max\{d_0 - UB, 0\}$ and $\min\{d_0 - LB, 0\}$. Given that the initial values do not depend on $\boldsymbol{\theta}$, the following holds for all $i = 0, ..., 3$

$$\frac{\partial}{\partial\theta_i}OG_0 = 0, \quad \frac{\partial}{\partial\theta_i}d_0 = 0, \quad \frac{\partial}{\partial\theta_i}\max\{d_0 - UB, 0\} = 0, \quad \frac{\partial}{\partial\theta_i}\max\{d_0 - LB, 0\} = 0.$$

Substituting these values into the partial derivatives of DP_1 results in

$$\begin{aligned}
\frac{\partial}{\partial\theta_0}DP_1 &= d_0, \\
\frac{\partial}{\partial\theta_1}DP_1 &= OG_0, \\
\frac{\partial}{\partial\theta_2}DP_1 &= \left(\frac{1+\hat{r}_1}{1+\hat{g}_1} - 1\right)d_0, \\
\frac{\partial}{\partial\theta_3}DP_1 &= (d_0 - 0.60).
\end{aligned}$$

For the next period, the same approach can be applied, leading to

$$
\begin{aligned}
\frac{\partial}{\partial\theta_0}DP_2 &= d_1 + \left(\theta_0 + \theta_2\left(\frac{1+\hat{r}_2}{1+\hat{g}_2} - 1\right) + \theta_3\right)\frac{\partial}{\partial\theta_0}d_1 + \theta_1\frac{\partial}{\partial\theta_0}OG_1 \\
&\quad + \frac{\partial}{\partial\theta_0}\max\{d_1 - UB, 0\} + \frac{\partial}{\partial\theta_0}\min\{d_1 - LB, 0\}, \\
\frac{\partial}{\partial\theta_1}DP_2 &= OG_1 + \left(\theta_0 + \theta_2\left(\frac{1+\hat{r}_2}{1+\hat{g}_2} - 1\right) + \theta_3\right)\frac{\partial}{\partial\theta_0}d_1 + \theta_1\frac{\partial}{\partial\theta_0}OG_1 \\
&\quad + \frac{\partial}{\partial\theta_0}\max\{d_1 - UB, 0\} + \frac{\partial}{\partial\theta_0}\min\{d_1 - LB, 0\}, \\
\frac{\partial}{\partial\theta_2}DP_2 &= \left(\frac{1+\hat{r}_2}{1+\hat{g}_2} - 1\right)d_1 + \left(\theta_0 + \theta_2\left(\frac{1+\hat{r}_2}{1+\hat{g}_2} - 1\right) + \theta_3\right)\frac{\partial}{\partial\theta_0}d_1 + \theta_1\frac{\partial}{\partial\theta_0}OG_1 \\
&\quad + \frac{\partial}{\partial\theta_0}\max\{d_1 - UB, 0\} + \frac{\partial}{\partial\theta_0}\min\{d_1 - LB, 0\}, \\
\frac{\partial}{\partial\theta_3}DP_2 &= (d_1 - 0.60) + \left(\theta_0 + \theta_2\left(\frac{1+\hat{r}_2}{1+\hat{g}_2} - 1\right) + \theta_3\right)\frac{\partial}{\partial\theta_0}d_1 + \theta_1\frac{\partial}{\partial\theta_0}OG_1 \\
&\quad + \frac{\partial}{\partial\theta_0}\max\{d_1 - UB, 0\} + \frac{\partial}{\partial\theta_0}\min\{d_1 - LB, 0\}.
\end{aligned}
$$

This again requires the partial derivatives of the auxiliary variables, which are given by

$$
\begin{aligned}
&\frac{\partial}{\partial\theta_i}OG_1 = \frac{1}{Y_1^p}\frac{\partial}{\partial\theta_i}Y_1, && \forall i = 0, 1, ..., 3, \\
&\frac{\partial}{\partial\theta_i}d_1 = \frac{1+r_1}{1+g_1}\frac{\partial}{\partial\theta_i}d_0 + d_0\frac{1}{(1+g_1)^2}\frac{\partial}{\partial\theta_i}g_1 - \frac{\partial}{\partial\theta_i}DP_1, && \forall i = 0, 1, ..., 3, \\
&\frac{\partial}{\partial\theta_i}\max\{d_1 - UB, 0\} = \begin{cases} \frac{\partial}{\partial\theta_i}d_1 & \text{if } d_1 > UB \\ 0 & \text{else} \end{cases}, && \forall i = 0, 1, ..., 3, \\
&\frac{\partial}{\partial\theta_i}\min\{d_1 - LB, 0\} = \begin{cases} \frac{\partial}{\partial\theta_i}d_1 & \text{if } d_1 < LB \\ 0 & \text{else} \end{cases}, && \forall i = 0, 1, ..., 3.
\end{aligned}
$$

Based on these derivations, it can be concluded that the partial derivatives of the auxiliary variables depend not only on the partial derivatives of the lagged variables (which are known) but also on the partial derivatives of Y_1 and g_1. The partial derivatives of Y_1 and g_1 are given by

$$
\begin{aligned}
&\frac{\partial}{\partial\theta_i}g_1 = c_1\frac{\partial}{\partial\theta_i}g_0 + c_2\frac{\partial}{\partial\theta_i}\Delta DP_1 - c_3\frac{\partial}{\partial\theta_i}OG_0, && \forall i = 0, 1, ..., 3, \\
&\frac{\partial}{\partial\theta_i}Y_1 = (1+g_0)\frac{\partial}{\partial\theta_i}Y_0 + Y_0\frac{\partial}{\partial\theta_i}g_0, && \forall i = 0, 1, ..., 3.
\end{aligned}
$$

Given that the partial derivatives of Y_0 and g_0 are zero (they do not depend on $\boldsymbol{\theta}$) partial derivatives of g_1 and Y_1 can be calculated, which in turn allows to calculate the partial derivatives of the other auxiliary variables. Note that the partial derivative of g_1 depends on the partial derivative of the discretionary policy in the same period (through the partial derivative of ΔDP_1). However, the partial derivative of g_1 is only required to calculate the partial derivative of DP_2, at which point the partial derivative of DP_1 is already known.

This procedure can be continued until the end of the timehorizon T, using the following recursive relations for the partial derivatives of DP_{t+1}

$$\frac{\partial}{\partial\theta_0}DP_{t+1} = d_t + (\theta_0 + \theta_2\left(\frac{1+\hat{r}_{t+1}}{1+\hat{g}_{t+1}} - 1\right) + \theta_3)\frac{\partial}{\partial\theta_0}d_t + \theta_1\frac{\partial}{\partial\theta_0}OG_t$$
$$+ \frac{\partial}{\partial\theta_0}\max\{d_t - UB, 0\} + \frac{\partial}{\partial\theta_0}\min\{d_t - LB, 0\}, \quad \forall t = 1, ..., T-1, \tag{36}$$

$$\frac{\partial}{\partial\theta_1}DP_{t+1} = OG_t + (\theta_0 + \theta_2\left(\frac{1+\hat{r}_{t+1}}{1+\hat{g}_{t+1}} - 1\right) + \theta_3)\frac{\partial}{\partial\theta_1}d_t + \theta_1\frac{\partial}{\partial\theta_1}OG_t$$
$$+ \frac{\partial}{\partial\theta_1}\max\{d_t - UB, 0\} + \frac{\partial}{\partial\theta_1}\min\{d_t - LB, 0\}, \quad \forall t = 1, ..., T-1, \tag{37}$$

$$\frac{\partial}{\partial\theta_2}DP_{t+1} = \left(\frac{1+\hat{r}_{t+1}}{1+\hat{g}_{t+1}} - 1\right)d_t + (\theta_0 + \theta_2\left(\frac{1+\hat{r}_{t+1}}{1+\hat{g}_{t+1}} - 1\right) + \theta_3)\frac{\partial}{\partial\theta_2}d_t + \theta_1\frac{\partial}{\partial\theta_2}OG_t$$
$$+ \frac{\partial}{\partial\theta_2}\max\{d_t - UB, 0\} + \frac{\partial}{\partial\theta_2}\min\{d_t - LB, 0\}, \quad \forall t = 1, ..., T-1, \tag{38}$$

$$\frac{\partial}{\partial\theta_3}DP_{t+1} = (d_t - 0.60) + (\theta_0 + \theta_2\left(\frac{1+\hat{r}_{t+1}}{1+\hat{g}_{t+1}} - 1\right) + \theta_3)\frac{\partial}{\partial\theta_3}d_t + \theta_1\frac{\partial}{\partial\theta_3}OG_t$$
$$+ \frac{\partial}{\partial\theta_3}\max\{d_t - UB, 0\} + \frac{\partial}{\partial\theta_3}\min\{d_t - LB, 0\}, \quad \forall t = 1, ..., T-1. \tag{39}$$

In addition, the recursive relations of the auxiliary variables are given by

$$\frac{\partial}{\partial\theta_i}g_{t+1} = c_1\frac{\partial}{\partial\theta_i}g_t + c_2\frac{\partial}{\partial\theta_i}\Delta DP_{t+1} - c_3\frac{\partial}{\partial\theta_i}OG_t, \quad \forall t = 1, ..., T-2, \tag{40}$$

$$\frac{\partial}{\partial\theta_i}Y_{t+1} = (1+g_t)\frac{\partial}{\partial\theta_i}Y_t + Y_t\frac{\partial}{\partial\theta_i}(g_t), \quad \forall t = 1, ..., T-2, \tag{41}$$

$$\frac{\partial}{\partial\theta_i}OG_{t+1} = \frac{1}{Y^p_{t+1}}\frac{\partial}{\partial\theta_i}Y_{t+1}, \quad \forall t = 1, ..., T-2, \tag{42}$$

$$\frac{\partial}{\partial\theta_i}d_{t+1} = \frac{1+r_{t+1}}{1+g_{t+1}}\frac{\partial}{\partial\theta_i}d_t + d_t\frac{1}{(1+g_{t+1})^2}\frac{\partial}{\partial\theta_i}g_{t+1} - \frac{\partial}{\partial\theta_i}DP_{t+1}, \quad \forall t = 1, ..., T-2, \tag{43}$$

$$\frac{\partial}{\partial\theta_i}\max\{d_{t+1} - UB, 0\} = \begin{cases} \frac{\partial}{\partial\theta_i}d_{t+1} & \text{if } d_{t+1} > UB \\ 0 & \text{else} \end{cases}, \quad \forall t = 1, ..., T-2, \tag{44}$$

$$\frac{\partial}{\partial\theta_i}\min\{d_{t+1} - LB, 0\} = \begin{cases} \frac{\partial}{\partial\theta_i}d_{t+1} & \text{if } d_{t+1} < LB \\ 0 & \text{else} \end{cases}, \quad \forall t = 1, ..., T-2. \tag{45}$$

Based on these recursive derivations, it can be concluded that all partial derivatives of DP_t can be calculated as long as the partial derivatives of DP_t and the auxiliary variables are stored during the construction of the path. Consequently, using the partial derivatives of $DP_1, ..., DP_T$, the IPA estimator can be calculated at the end of the simulation run. This suggests that, in addition to the FD estimator, the IPA estimator can also be employed in this analysis.

In contrast to the FD estimator, the IPA estimator shows how the individual partial derivatives are affected by the different components embedded in the model. Analysing the recursive partial derivatives of DP_t, it can be observed that the similarities are substantial. The largest difference is discovered in the first component of the partial derivatives in (36 – 39). Given this discrepancy, it is expected that the partial derivative of the objective function with respect to θ_0 is characterised by a substantially higher level of volatility than the other coefficients. This can be explained by the fact that debt ratios comprise a larger range of values than the output gap, the excess debt value, and the expected change in debt ratio. For θ_3 applies the same, however, to a lesser extent.

In contrast to the FD estimator, this IPA estimator should theoretically be unbiased. For the analysis, the FD estimator was selected due to its easy implementation. Additionally, as c_k decreases in the limit, the FD estimator approaches the partial derivatives obtained by the IPA estimator. Hence, theoretically, both estimators should lead to the same optimisation result.

4 Methodology

This section will begin by outlining the model used in the analysis. Subsequently, the evaluation metrics for the various scenarios will be displayed, and the data used will be discussed.

4.1 Model description

The algorithm employed in this analysis to project the development of government debt follows (4). In addition, projections of the underlying macroeconomic variables will follow (5 - 7) for growth rates, interest rates, and primary balance, respectively. Algorithm 1 summarises the complete procedure for the simulation of debt ratios. The procedure requires distributions for interest rate shocks, growth shocks, and primary balance shocks (F_r, F_g, F_{pb}). Moreover, the time horizon (T) to project the debt ratios over must be predefined, which is set at $T = 40$ in this analysis. The initiation of the algorithm requires a starting value for the debt ratio, output gap, growth rate, interest rate, and GDP. In this analysis, the starting year for the projections is 2022, which provides all the starting values. The algorithm proceeds as follows, first shocks are simulated using the predetermined distributions. Consequently, the interest rate, primary balance, and the growth rate are calculated. The adjustment term of the primary balance (DP_t) depends on the scenario, which is determined using Algorithm 2. If the debt constraint is imposed (indicated with a boolean value for *debt constraint*, the constraint procedure described in Algorithm 3 is called. Lastly, the procedure updates the debt ratio and GDP using the simulated underlying macroeconomic variables.

Algorithm 1 DSA simulation

Require: F_g, F_r, F_{pb}, time-horizon T

procedure DSASIMULATION
    Init:
    $d_0 \leftarrow$ start value debt ratio
    $OG_0 \leftarrow$ start value output gap
    $g_0 \leftarrow$ start value growth rate
    $r_0 \leftarrow$ start value interest rate
    $Y_0 \leftarrow$ start value GDP

    for $t = 1$ **to** T **do**
        Determine shocks
        $\omega_t \sim F_g$ ▷ Growth shock
        $i_t \sim F_r$ ▷ Interest shock
        $\epsilon_t \sim F_{pb}$ ▷ Primary balance shock
        Determine interest rate
        $r_t \leftarrow \max\{0, r_{t-1} + i_t\}$
        Determine primary balance
        $DP_t \leftarrow$ DETERMINEDISCRETIONARYPOLICY(t)
        $pb_t \leftarrow DP_t + \pi_t + \beta\omega_t + \epsilon_t$ ▷ π_t is predetermined
        if *debt constraint* **then**
            Call CHECKDEBTRATIOCONSTRAINT(d_{t-1}, DP_t, pb_t)
        end if
        Determine growth rate
        $\Delta DP_t \leftarrow DP_t - DP_{t-1}$
        $OG_{t-1} \leftarrow \frac{Y_{t-1} - Y^p_{t-1}}{Y^p_{t-1}}$ ▷ Y^p_t is predetermined
        $g_t \leftarrow c_1 g_{t-1} + (1 - c_1) g^p_t + c_2 \Delta DP_t - c_3 OG_{t-1} + \omega_t$ ▷ g^p_t is predetermined

        Update values
        $d_t \leftarrow \frac{1+r_t}{1+g_t} d_{t-1} - pb_t$
        $Y_t \leftarrow (1 + g_t) Y_{t-1}$
    end for
end procedure

Algorithm 2 Debt ratio constraint

function DETERMINEDISCRETIONARYPOLICY(t)
    if Baseline scenario **then**
        $DP_t \leftarrow 0$
    else if Debt constraint **then**
        $DP_t \leftarrow \max\{d_{t-1} - UB, 0\}$
    else if Deficit constraint **then**
        $DP_t \leftarrow \max\{\underline{PB} - \pi_t - \beta\omega_t - \epsilon_t + \hat{R}_t, 0\}$
    else if Neutral debt target **then**
        $DP_t \leftarrow \left(\frac{1+\hat{r}_t}{1+\hat{g}_t} - 1\right) d_{t-1} - \pi_t$
    else if Explicit debt target **then**
        $\xi_t \leftarrow (d_{t-1} - 0.60)/20$
        $DP_t \leftarrow \left(\frac{1+\hat{r}_t}{1+\hat{g}_t} - 1\right) d_{t-1} + \xi_t - \pi_t$
    else if Optimisation **then**
        $DP_t \leftarrow \theta_0 d_{t-1} + \theta_1 OG_{t-1} + \theta_2 \left(\frac{1+\hat{r}_t}{1+\hat{g}_t} - 1\right) d_{t-1} + \theta_3(d_{t-1} - 0.60) - \pi_t$
    end if
end function

Algorithm 3 describes the procedure used when a constraint is imposed on the debt ratio, which requires a lower and upper bound on the debt ratio (default values are $LB = -\infty$, $UB = \infty$). The procedure evaluates the debt ratio at the end of the previous period and adjusts the primary balance if necessary by adjusting the DP_t. However, a change in DP_t also affects the primary balance pb_t. As such, the primary balance is also adjusted if the constraint is violated.

Algorithm 3 Debt ratio constraint

Require: Lowerbound LB, Upperbound UB
procedure CHECKDEBTRATIOCONSTRAINT(d_{t-1}, DP_t, pb_t)
    if $d_{t-1} > UB$ **then**
        $adj_t \leftarrow UB - d_{t-1}$
        $DP_t \leftarrow DP_t - adj_t$
        $pb_t \leftarrow pb_t - adj_t$
    else if $d_{t-1} < LB$ **then**
        $adj_t \leftarrow LB - d_{t-1}$
        $DP_t \leftarrow DP_t - adj_t$
        $pb_t \leftarrow pb_t - adj_t$
    end if
end procedure

The objective of the analysis is to optimise the primary balance using simulated debt ratios, with an emphasis on the explicit adjustment (DP_t) of the primary balance. In the optimisation problem, the primary balance is given as a function of $\boldsymbol{\theta}$, which represents a vector of coefficients of the components of the adjustment term. A summary of the algorithm used to find the minimum of the objective function (23) is given in Algorithm 4. The procedure requires an initial $\boldsymbol{\theta}$, a stepsize sequence ϵ_k, a stopping rule, and a perturbation stepsize sequence c_k. Based on a preliminary analysis, it was found that the perturbation stepsize sequence of the FD estimator and the gain sequence must be chosen cautiously. Besides the fact that large stepsizes may lead to overshooting and induce a high level of volatility, it also causes the model to explode and/or implode due to the endogenous relations embedded in the model. A large (perturbation) stepsize may result in large disruptions in DP_t, which affects the growth simulation and causes exploding and/or imploding debt ratios. As such, we must choose a sufficiently small step size. However, a small step size may obstruct a fast convergence.

The optimisation procedure used in this analysis consists of three parts. First, there will be a simultaneous optimisation of all coefficients in $\boldsymbol{\theta}$. After the first phase, the algorithm continues to optimise θ_2. In a preliminary analysis, it was found that θ_2 has more difficulty converging to its optimum. To accelerate the optimisation process, the other coefficients are fixed at the average estimate of the last 50 iterations. An analysis of the gradient indicated that the other coefficients cause large disturbances, which interfere with the optimisation process of θ_2, see Appendix B. As such, θ_2 is updated individually during phase 2. Finally, the full vector of estimates will be optimised simultaneously with a smaller step size for some final adjustments. The different phases are divided by K_1, K_2, and K_3. Values for the stopping rule of each phase are chosen on the basis of convergence results in the preliminary analysis. After $K_1 = 20,000$ iterations, all coefficients appeared to be converged, except for θ_2 for which the estimate continued to exhibit minor fluctuations. Therefore, phase 2 commences and continues until the total number of iterations has reached $K_2 = 40,000$, which implies that phase 2 also consists of $20,000$ iterations. This number of iterations is necessary for θ_2 to stabilise without the noise of the other coefficients. Lastly, phase 3 concludes the optimisation process and continues for a total of $K_3 = 50,000$ iterations. Although this may seem quite large, allowing for another $10,000$ iterations ensures that the estimates are almost surely converged.

In the procedure, it was chosen to employ a constant epsilon function ($\epsilon_t = \epsilon$), which starts at $\epsilon = 0.05$ and reduces to $\epsilon = 0.01$ in the final part of the optimisation process. Preliminary analysis indicated that the coefficients remain relatively small; hence, implementing a large step size results in unnecessary overshooting. In addition, the size of the perturbation step is defined as $c_k = 1/(1000 + k)^{1/2}$. This definition ensures that the perturbation remains relatively small and decreases gradually afterwards. Larger perturbations were found to simply induce more noise and may even disrupt the optimisation process. Furthermore, the optimisation procedure starts with $\boldsymbol{\theta}_0 = [0, 0, 0, 0]$ which should theoretically be the optimal solution when no constraints are imposed. That is, when all coefficients are zero, $DP_t = 0, \forall t = 1, ..., T$ and $\Delta DP_t = 0, \forall t = 2, ..., T$, which leads to a standard deviation of zero.

Due to the simple linear definition of the adjustment term and the mostly linear relations embedded in the model, it can be concluded that the gradient should find an optimum theoretically. In addition, the linear structure allows for a very small perturbation stepsize as we do not have to consider any discontinuities in the objective function.

Algorithm 4 Optimization algorithm

Require: Initial value $\boldsymbol{\theta}_0$, stopping rule for each phase K_1, K_2, K_3

procedure OPTIMISATIONALGORITHM
    for $k = 1$ **to** K_1 **do**
        $c_k \leftarrow 1/(1000 + k)^{1/2}$
        $\epsilon \leftarrow 0.05$
        $\mathbf{g} \leftarrow$ CALCULATEGRADIENT$(c_k, \theta_\mathbf{k})$
        $\boldsymbol{\theta}_k \leftarrow \boldsymbol{\theta}_{k-1} - \epsilon \mathbf{g}$
    end for
    $\boldsymbol{\theta}_k \leftarrow \frac{1}{50} \sum_{i=K_1-50}^{K_1} \boldsymbol{\theta}_i$
    for $k = K_1 + 1$ **to** K_2 **do**
        $c_k \leftarrow 1/(1000 + k)^{1/2}$
        $\epsilon \leftarrow 0.05$
        $\mathbf{g} \leftarrow$ CALCULATEINDIVIDUALGRADIENT$(c_k, \theta_\mathbf{k})$
        $\boldsymbol{\theta}_k \leftarrow \boldsymbol{\theta}_{k-1} - \epsilon \mathbf{g}$
    end for
    $\boldsymbol{\theta}_k \leftarrow \frac{1}{50} \sum_{i=K_2-50}^{K_2} \boldsymbol{\theta}_i$
    for $k = K_2 + 1$ **to** K_3 **do**
        $c_k \leftarrow 1/(1000 + k)^{1/2}$
        $\epsilon \leftarrow 0.01$
        $\mathbf{g} \leftarrow$ CALCULATEGRADIENT$(c_k, \theta_\mathbf{k})$
        $\boldsymbol{\theta}_k \leftarrow \boldsymbol{\theta}_{k-1} - \epsilon \mathbf{g}$
    end for
    return $\frac{1}{50} \sum_{i=K_3-50}^{K_3} \boldsymbol{\theta}_i$ ▷ Average of last 50 iterations
end procedure

The gradient will be calculated using the FD estimator, which is given in Algorithm 5. In this procedure, $\mathbf{e_i}$ represents a basic vector with components $x_i = 1$ and $x_n = 0$ for all $n \neq i$. The gradient estimator uses *gradient clipping*, which prevents the gradient from exploding. For the clipping of gradients, a threshold of 0.5 was chosen (based on the gradient histogram from a preliminary analysis; see Appendix C). Furthermore, $\|\cdot\|$ represents the Euclidean norm. In addition, the second phase of the optimisation procedures uses CALCULATEINDIVIDUALGRADIENT(c_k, θ_k) to proceed with the individual optimisation of θ_2. The gradient estimator uses the objective function, given in (23), which is calculated using Algorithm 6.

Algorithm 5 Gradient estimator

Require: Threshold $\overline{c}$

function CALCULATEGRADIENT(c_k, $\boldsymbol{\theta}_k$)
  for $i = 0, ..., 3$ **do**
    $J(\boldsymbol{\theta}^{+}) \leftarrow$ CALCULATEOBJECTIVEFUNCTION($\boldsymbol{\theta} + c_k \mathbf{e_i}$)
    $J(\boldsymbol{\theta}^{-}) \leftarrow$ CALCULATEOBJECTIVEFUNCTION($\boldsymbol{\theta} + c_k \mathbf{e_i}$)
    $g_i \leftarrow \frac{J(\boldsymbol{\theta}^{+}) - J(\boldsymbol{\theta}^{-})}{2c_k}$
  end for
  if $\|\mathrm{g}\| > \overline{c}$ **then**
    $\mathbf{g} \leftarrow \overline{c}\frac{\mathbf{g}}{\|\mathbf{g}\|}$
  end if
  $\mathbf{g} \leftarrow [g_0, g_1, g_2, g_3]$
  return g
end function

function CALCULATEINDIVIDUALGRADIENT(c_k, $\boldsymbol{\theta}_k$)
  $J(\boldsymbol{\theta}^{+}) \leftarrow$ CALCULATEOBJECTIVEFUNCTION($\boldsymbol{\theta} + c_k \mathbf{e_2}$)
  $J(\boldsymbol{\theta}^{-}) \leftarrow$ CALCULATEOBJECTIVEFUNCTION($\boldsymbol{\theta} + c_k \mathbf{e_2}$)
  $g_2 \leftarrow \frac{J(\boldsymbol{\theta}^{+}) - J(\boldsymbol{\theta}^{-})}{2c_k}$
  $\mathbf{g} \leftarrow [0, 0, g_2, 0]$
  return g
end function

Algorithm 6 Objective function

function CALCULATEOBJECTIVEFUNCTION($\boldsymbol{\theta}$)
  Run DSASIMULATION to obtain $[\Delta DP(\boldsymbol{\theta})_1 \cdots \Delta DP(\boldsymbol{\theta})_T]$
  $\overline{DP} \leftarrow \frac{1}{N}\sum_{t=1}^{T} \Delta DP(\boldsymbol{\theta})_t$
  $J(\boldsymbol{\theta}) \leftarrow \sqrt{\frac{\sum_{t=2}^{T}(\Delta DP(\boldsymbol{\theta})_t - \overline{DP})^2}{T-2}}$
  return $J(\boldsymbol{\theta})$
end function

4.2 Evaluation measures

In order to compare the (benchmark) scenarios and the optimisation results, different evaluation measures will be employed. As a measure of stability, this analysis will take into account the volatility of fiscal policy, proxied by the standard deviation of the changes in explicit adjustment (ΔDP_t) of primary balance, given by

$$\sigma_{\Delta DP} = \sqrt{\frac{\sum_{t=2}^{T}\left(\Delta DP_t - \overline{\Delta DP}\right)^2}{T-2}}, \tag{46}$$

in which $\Delta DP_t = DP_t - DP_{t-1}, \forall t = 2, ..., T$ and $\overline{\Delta DP}$ is calculated using (24). The change in the adjustment term indicates whether additional contractions ($\Delta DP_t > 0$) or expansions ($\Delta DP_t < 0$) must be implemented with respect to the adjustment term in the previous period. This can be explained by the fact that a larger primary deficit leaves more space for additional expenses and/or reduced revenues. Volatility in contractions and/or expansions causes macroeconomic instability. As such, it is preferred to minimise volatility.

In addition, to measure the degree of countercyclicality, it is important to understand the relationship between additional policy interventions and the state of the economy. This analysis relies on the assumption that additional policy interventions are represented by a change in the adjustment term DP_t. Furthermore, the state of the economy can be measured by using the output gap. A positive output gap corresponds to economic boom periods, whereas a negative output gap is often related to periods of contraction. Using these concepts, the countercylical efficiency will be measured using a simple linear regression given by

$$\Delta DP_t = \alpha + \gamma OG_t. \tag{47}$$

The coefficient of interest in this equation is γ, which should be positive for the policy to be countercyclical. A positive coefficient indicates a positive relationship between the output gap and the change in the primary balance, i.e., a positive output gap (economic boom period) is responded to with additional fiscal contraction. Implementing contractionary policy during times of economic growth is an example of countercyclical policy.

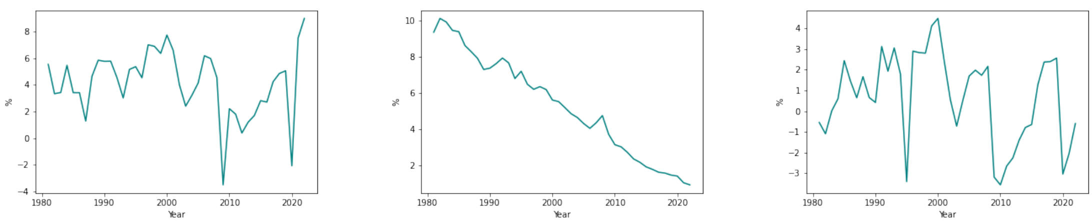
Lastly, the aim is to avoid exploding debt ratios and minimise the risk of default. While the literature reveals different limits for debt to remain sustainable, all above 100% of GDP for the Netherlands (see, for example, Fournier and Fall (2015) and Ghosh, Kim, et al. (2013)), a rather conservative approach with 75% as indicator was chosen. This indicator can be seen as an early warning of rising debt. This indicator takes into account the fact that there is often a lack of political support for high debt ratios. The reported measure of exploding debt will be the probability of debt exceeding the 75% level at the end of the time horizon, given by

$$\frac{1}{N}\sum_{i=1}^{N} \mathbb{I}_{d_{i,T}>0.75}, \qquad \mathbb{I}_{d_{i,T}>0.75} = \begin{cases} 1 & \text{if } d_{i,T} > 0.75 \\ 0 & \text{else} \end{cases}, \forall i = 1, ..., N, \tag{48}$$

in which $\mathbb{I}_{d_{i,T}>0.75}$ indicates whether the debt ratio has surpassed the 75% level at the end of the time horizon T during iteration i.

4.3 Data

The data used in this analysis are retrieved from the *Macro Economische Verkenning 2023 (MEV)* of the Bureau for Economic Policy Analysis (CPB). This data set contains historical data on growth, interest payments, inflation, primary balances, and historical debt ratios. The model is based on nominal terms; as such, all variables are converted to nominal terms if necessary. For this purpose, the real index numbers were multiplied by the corresponding inflation index number. The data range from 1981 to 2022. Figure 1 reveals the historical development of macroeconomic variables during the period 1981-2022.

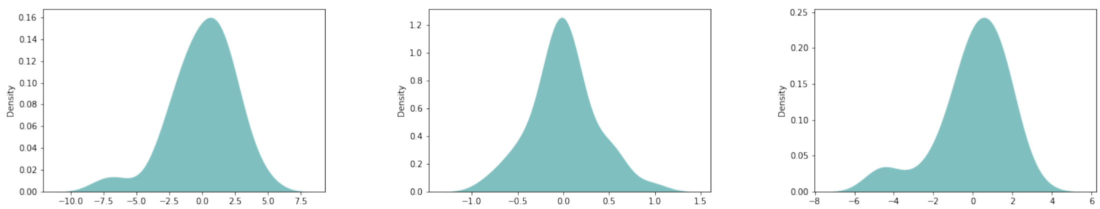


(a) Growth rates (b) Interest rates (c) Primary balances

Figure 1: Historical development

4.3.1 Distributions

In the analysis, the shocks are drawn from the historical distribution of the shocks. Historical growth shocks are obtained by demeaning historical nominal growth rates. For the interest rate, the historical distribution of interest innovations will be employed ($\tilde{r_t} = r_t - r_{t-1}, \forall t = 1982, ..., 2022$), which is also centred on an average of zero. Lastly, the primary balance shocks are drawn from the residuals of the linear regression of primary balance on growth shocks, $pb_t = \alpha + \beta\omega_t$, with $\beta = 0.60$ (in line with Mourre et al. (2014)). Using historical distributions allows us to incorporate historical volatility to model the future. Figure 2 shows the historical distributions of the innovations of interest, growth shocks, and primary balance residuals. The distributions suggest that the shocks are not normally distributed, which also encourages the use of historical distributions.



(a) Growth shocks (b) Interest innovations (c) Primary balance shocks

Figure 2: Distribution of the historical shocks

4.3.2 Assumptions

Besides historical data, the model also relies on assumptions about the future behaviour of (potential) growth and primary balance. First, in the baseline model, it is assumed that the level of potential growth remains constant at 4% throughout the entire time horizon, that is, $g_t^p = 4\%, \forall t = 1, ..., T$. This is based on 2% real growth, which is the historical average of the Netherlands, and 2% expected inflation, which is the aim of the ECB (DNB, n.d.). In addition, potential GDP is modelled recursively in this model using

$$Y_t^p = (1 + g_t^p)Y_{t-1}^P, \qquad \forall t = 1, ..., T, \tag{49}$$

$$Y_0^P = \frac{Y_0}{1 + OG_0}. \tag{50}$$

Hence, potential GDP (Y_t^P) is calculated by multiplying the potential GDP in the previous period (Y_{t-1}^p) by the potential growth factor $(1 + g_t^p)$. Furthermore, the initial potential GDP (Y_0^p) is calculated using the definition of the output gap ($OG_t = (Y_t - Y_t^p)/Y_t^P, \forall t = 1, ..., T$) and the estimated output gap of 2022 as the starting value of the potential GDP (Y_0^p). The estimated output gap is retrieved from the *AMECO* dataset provided by the *European Commission*. Lastly, the model employed in this analysis uses a basic path for the primary balance of -1.75% of GDP, which is in line with the last coalition agreement (CPB, n.d.).

In addition to the baseline assumption, this analysis will also look at the optimisation of fiscal policy under different circumstances. Given the large degree of uncertainty surrounding the development of macroeconomic variables, it is also interesting to look at different (potential) growth expectations, starting interest rates, and other assumptions regarding the basic path of the primary balance. Exploring various assumptions can provide insight into how to respond when the economic environment changes. Hence, for each of these elements, this analysis will look at a more optimistic and more pessimistic scenario, using a ceteris paribus assumption (i.e., other assumptions remain constant). For growth, a more pessimistic scenario entails a reduced potential growth rate, whereas for the interest rate, the pessimistic scenario is characterised by a higher rate. In addition, for the primary balance, it is assumed that the pessimistic scenario is represented by a lower basic path of the primary balance (i.e., larger primary deficit).

5 Results

Section 5 contains the results of the analysis. The first section describes the evolution of debt in the benchmark models. The second section proceeds to the optimisation results and compares these results with the benchmark scenarios. Lastly, the impact of different assumptions will be tested.

5.1 Evaluation benchmark scenarios

Simulations of benchmark scenarios clearly show the impact of different fiscal policies. First, the baseline scenario clearly demonstrates the enormous uncertainty surrounding the development of the debt ratio. Figure 3 depicts the simulation results of the $n = 10.000$ iterations. In this figure, the mean value is depicted by the orange line and the shaded areas provide 5% percentile intervals of the simulation results, ranging from 5% to 95%. The results indicate that the 95% confidence interval at the end of the time horizon ($T = 40$) ranges from 41.07 to 104.94. On the basis of these results, it can be concluded that, in the most favourable circumstances, the debt ratio can be reduced to a mere 41% of GDP. However, in the majority of the iterations, debt ratios greater than 60% are achieved (median = 61.12 in $t = 40$). Furthermore, the worst scenarios are characterised by debt ratios greater than 100% of GDP.

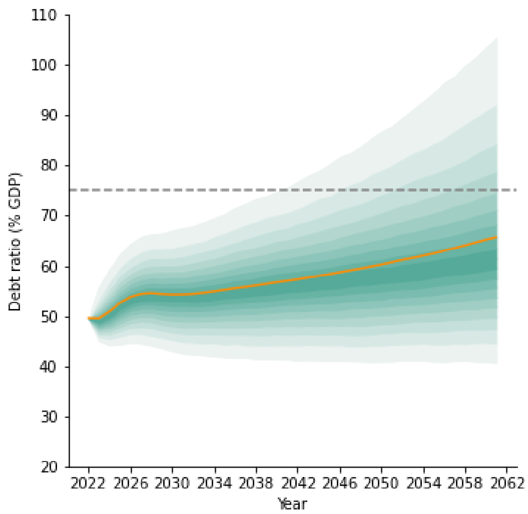


Figure 3: Fan plot of the baseline simulation

In addition to the baseline scenarios, projections are created of other benchmark scenarios. Figure 4 reveals the development of the debt ratio for these different scenarios and the corresponding development of the adjustment term DP_t. Additionally, each scenario is evaluated using three measures to determine the level of volatility in the primary balance, countercyclical efficiency, and default risk; the results can be found in Table 1.

First, it should be noted that the implementation of ad hoc constraints has profound consequences on the development of the debt ratio and the volatility of the primary balance. Figure 4a and 4b indicate that both constraints prevent the debt ratio from exploding. The evaluation measures in Table 1 indicate that imposing restrictions lead to high volatility in explicit policy adjustments (see Figure 4e and 4f). Interpreting the evaluation measures, it can be concluded that for the deficit constraint, on average, the standard deviation of the change in discretionary policy is remarkable 1.841% of GDP. This indicates a very high level of volatilitity in contractions and expansions. Furthermore, the countercyclical efficiency indicator suggests that policy adjustments as a result of the implementation of constraints tend to have a procyclical influence. The countercyclical efficiency coefficient can be interpreted as the change in the discretionary policy of the primary balance as a result of a change in the output gap. Using this definition, it can be concluded that a 1% change in the output gap leads to a fiscal expansion of 0.03% of GDP for the deficit constraint (due to a negative change in discretionary policy, i.e. $\Delta DP_t < 0$). Economically, this means that economic boom periods are correlated with fiscal expansion, which is known as procyclical policy.

Compared to ad hoc restrictions, the neutral debt goal and the debt goal of 60% appear to have a more moderate effect on the evolution of debt ratios and discretionary policy, which can be seen in the right two columns of Figure 4. The level of uncertainty in the neutral debt target remains relatively high, with debt ratios ranging between 23% and 79% of GDP. Moreover, the discretionary policy necessary to aim for a neutral debt target appears to be slightly countercyclical, while the adjustment of the explicit debt target is slightly procyclical.

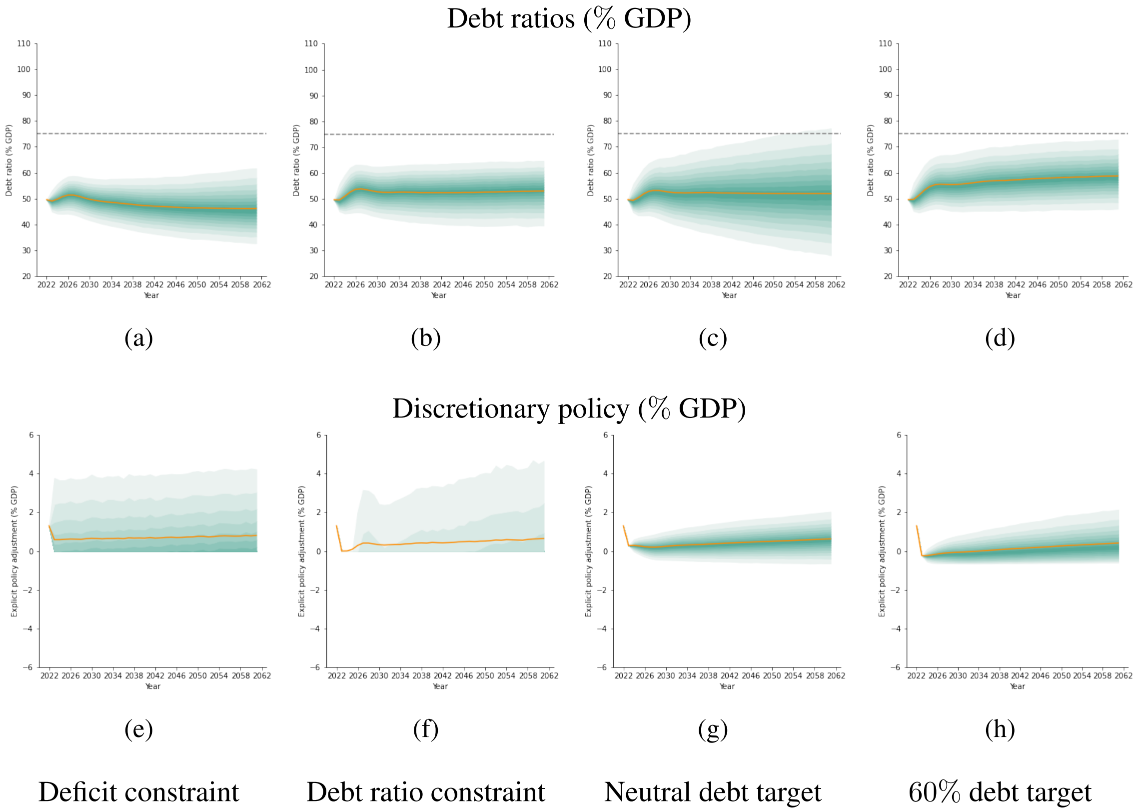


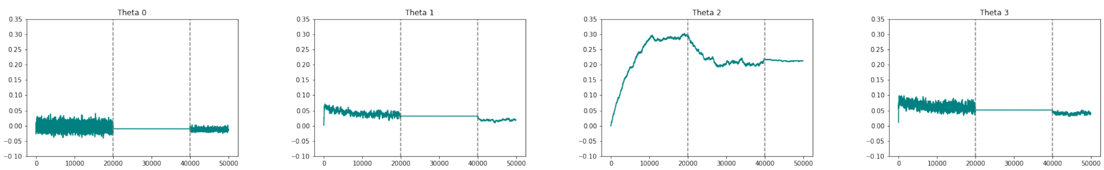
Figure 4: Results Benchmark Scenarios

Table 1: Evaluation measures of the benchmark scenarios

Measure	**Deficit constraint**	**Debt ratio constraint**	**Neutral debt target**	**Explicit debt target**
Volatility policy	1.847	1.294	0.184	0.202
Countercyclicality	-0.030	-0.069	0.012	-0.012
Default risk	0.001	0.001	0.066	0.027

5.2 Optimisation results

Using Algorithm 4, optimal coefficients of $\boldsymbol{\theta}$ are obtained for the discretionary policy function given in (51). In addition, constraints are imposed on the debt ratio such that $d_t \in [30\%, 80\%]$. The optimisation procedure was run 30 times to create a density of the distribution of $\boldsymbol{\theta}$, see Appendix D for a detailed analysis of the estimates obtained in the optimisation procedure. This meta-analysis suggests that the coefficients are likely to be normally distributed and characterised by a relatively small standard deviation. Figure 5 shows the results of the stochastic optimisation procedure used to determine the optimal value of $\boldsymbol{\theta}$. The vertical dashed grey lines indicate the difference phases in the optimisation process. Figure 5 clearly shows the fast convergence of θ_0, θ_1, and θ_3. Moreover, Figure 5c indicates that fixing the other coefficients clearly improved the convergence process of θ_2. In addition, including the other coefficients in the last phase does not seem to affect the estimation of θ_2, nor does it seem to cause large changes in the estimates of θ_0, θ_1 and θ_3.



(a) (b) (c) (d)

Figure 5: Optimisation results

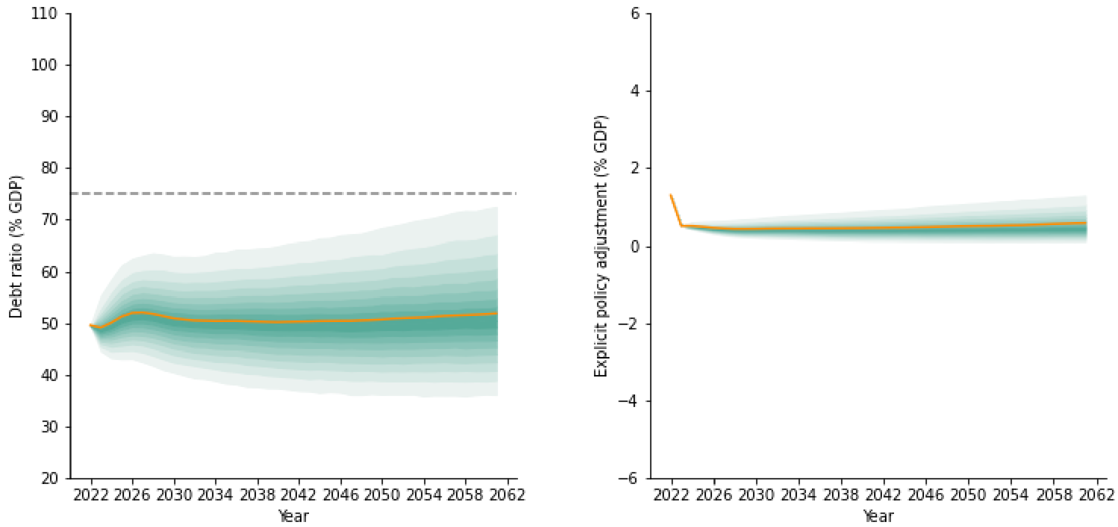
Based on the optimisation process, estimates of the coefficients are obtained by the mean value of the last 50 iterations of the optimisation algorithm, which has resulted in the following equation for structural primary balance

$$DP(\theta)_t = \theta_0 d_{t-1} + \theta_1 OG_{t-1} + \theta_2 \left(\frac{1+\hat{r}_t}{1+\hat{g}_t} - 1 \right) d_{t-1} + \theta_3 (d_{t-1} - 0.60) \quad (51)$$

$$= -0.0126\, d_{t-1} + 0.019\, OG_{t-1} + 0.213 \left(\frac{1+\hat{r}_t}{1+\hat{g}_t} - 1 \right) d_{t-1} + 0.041\, (d_{t-1} - 0.60). \quad (52)$$

Interestingly, the optimisation results indicate that only part of the expected change in the debt ratio is reversed as $\theta_2 < 1$. A full reversion would require $\theta_2 = 1$, similar to the neutral debt target scenario. Furthermore, the adjustment term suggests that, in the optimal scenario, the excess debt ratio should be reduced by approximately $1/24$ every year, rather than $1/20$. The output gap seems to have a positive effect on discretionary policy, indicating that during cyclical upturns, discretionary policy enhances the primary balance, which is associated with reduced expenses and/or government increased revenue). Lastly, the influence of the individual lagged debt coefficient d_{t-1} can be obtained by combining θ_0 and θ_2 resulting in a coefficient of 0.2004. This suggests a positive relationship between the debt ratio and the adjustment term.

Figure 6a depicts the results of the DSA simulation of the optimised fiscal policy rule. The debt ratio appears to be stabilising over time on average, and the uncertainty around the average seems to be of moderate size, yet slightly skewed to the right. Furthermore, in Figure 6b it can be observed that the distribution of the discretionary policy term remains relatively small, indicating a low level of volatility in additional contractionary and expansionary policy.



(a) Debt ratios

(b) Discretionary policy

Figure 6: Fan charts of the optimised scenario

Table 2 shows the evaluation measures of the optimised scenario. It can be denoted that the optimised scenario clearly outperforms the neutral and explicit debt target scenario based on the volatility in policy. The average standard deviation per run was found to be 0.118% of GDP, which is a reduction of approximately 35% compared to the neutral debt target. Moreover, countercyclical efficiency indicates that the fiscal policy maintained in the optimal scenarios is acyclical, implying little to no interaction between fiscal policy and the state of the economy. The probability of an explosion of debt is 3.7%.

Table 2: Evaluation measures of the optimised scenarios

Measure	**Neutral debt target**	**Explicit debt target**	**Optimised scenario**
Volatility policy	0.184	0.202	0.118
Countercyclicality	0.012	-0.012	0.001
Default risk	0.066	0.027	0.037

5.2.1 Comparison optimisation results

In order to compare the optimised scenario with the benchmark scenarios, an *ordinary least squares* (OLS) regression can be performed on the simulated structural primary balances. Using a *seed* to establish a fair comparison, the discretionary policy (DP_t) is simulated for each scenario over $n = 10.000$ runs. Consequently, an OLS regression was performed with the structural primary balance of the optimised scenarios as dependent variable and the structural primary balance of the other variables as independent variables. Table 3 provides an overview of the regression results, indicating that the neutral debt target has strong explanatory power. Note that the neutral debt target and the explicit debt target are highly correlated, as such, the explicit debt target is omitted from the model. Furthermore, the explanatory power of the combined benchmark scenarios was moderate ($R^2 = 0.365$).

Table 3: OLS results to compare scenarios

	OLS
Deficit constraint	0.007^* (0.000)
Debt ratio constraint	0.010^* (0.000)
Neutral debt target	0.666^* (0.001)

Note: Standard Errors in parentheses; $^*p < 0.01$

5.2.2 Different assumptions

Table 4 reveals the optimisation results of different assumptions. First, the low growth scenario uses a more pessimistic potential growth of 2.9%, while in the high growth scenario, growth is expected to stabilise around 5%. For the assumption of the starting value of the interest rate, the low interest scenario starts at the zero lower bound, which mirrors a more optimistic scenario in which interest rates remain low. The pessimistic scenario is characterised by a start interest rate of 3%. Lastly, for the primary balance scenarios, a base path of -3% is used for the pessimistic scenario (denoted by *Low*) and -1% for the optimistic scenario.

The estimations of the coefficients indicate that in pessimistic scenarios (i.e., low growth, high interest rates, and low primary balance), θ_2 increases significantly compared to the baseline estimate of $\theta_{B,2} = 0.213$. Furthermore, in the high growth scenario and the high primary balance scenario, an increase in θ_2 can also be observed, which implies that a larger part of the expected change is reversed. This effect is particularly visible in scenarios in which the difference between the growth rate and the interest rate is large. The changes in θ_0 seem relatively small for all scenarios; Nevertheless, the initial estimate in the baseline scenario is already quite small, so the relative changes compared to the baseline optimisation model can still be considerable. It is noteworthy that the coefficient of the output gap seems to gain significance for several scenarios. Compared to the estimate in the baseline scenario ($\theta_{B,1} = 0.019$), we can observe a large increase for growth scenarios, the high interest scenario, and the primary balance scenarios. Lastly, it can be observed that the value of θ_3 is only significantly affected by alterations in the assumed interest rate.

In addition to the estimates of the coefficients ($\theta_i, i = 0, ..., 3$), Table 4 also presents the evaluation measures obtained from the DSA simulation of the adjustment term containing the optimised coefficients. It can be immediately observed that, for all scenarios, except the low interest scenario, a higher level of volatility is reported, which means that if either growth, interest, or the base path of the primary balance deviates from the baseline assumption, a higher level of intervention is required. Furthermore, it can be observed that, in most scenarios, the optimised fiscal policy remains acyclical, slightly tending to countercyclical in some scenarios. The chance that debt ratios become unsustainable seems only to increase in the *low* primary balance scenario.

Table 4: Optimisation results of different assumptions

	Growth		Interest		Primary balance	
	Low	**High**	**Low**	**High**	**Low**	**High**
θ_0	0.008	-0.008	-0.017	0.001	-0.005	-0.002
θ_1	0.038	0.027	0.012	0.032	0.032	0.031
θ_2	0.410	0.313	0.218	0.395	0.334	0.341
θ_3	0.043	0.046	0.035	0.038	0.044	0.042
Volatility policy	0.170	0.137	0.098	0.176	0.197	0.162
Countercyclicality	0.012	0.001	-0.000	0.007	0.011	-0.002
Default risk	0.018	0.032	0.037	0.037	0.056	0.034

6 Discussion

The results of the benchmark scenarios underscore the importance of long-term vision for fiscal stability. The introduction of ad hoc constraints had a significant effect on the variability of fiscal policy, while the implementation of a long-term anchor gradually altered the debt ratio, leading to a much more stable situation. These results are in line with Barro (1979) and Bhandari et al. (2017), who concluded from a welfare point of view that, in the optimal scenario, the government should aim to maintain a long-term debt target. Additionally, while the neutral debt target scenario is still characterised by a considerable amount of uncertainty in the development of the debt ratio, adding a small adjustment term seems to reduce this uncertainty significantly without compromising the low volatility in the adjustment term. However, it has a small procyclical impact on the discretionary policy.

Although the neutral debt target can be seen as a significant improvement compared to the ad hoc constraints and the baseline scenario without any policy intervention, it does not fully control exploding debt ratios. Implementations of ad hoc constraints and the explicit debt target seem to resolve this problem; however, these interventions cause a rise in volatility. Based on these results, an optimal combination was found which confines the debt ratio without causing large disruptions in the government balance. Using the optimised fiscal policy, which can be seen as an adaptation of the neutral and explicit debt target, a consistent fiscal policy can be maintained that remains within the allowed debt space. Although the risk of exploding debt is not completely eliminated, it can be seen as a significant improvement compared to the neutral debt target. To minimise the risk of default, stricter restrictions can be imposed on the debt ratio in the optimisation process. In this analysis, constraints are imposed such that the debt ratio must remain above 30% and below 80%.

Interestingly, the optimisation results reveal that the impact of the term reversing the expected change in debt is relatively small. Whereas in the neutral debt target scenario, the full expected change is reversed, in the optimised scenario, approximately $^1/_5$-th of the expected change is compensated. This may imply that the reversal implemented in the neutral debt ratio, used to maintain a consistent debt ratio in expectation (as suggested by Barro (1979)) may be too invasive. Furthermore, the optimisation results indicate that rather than diminishing excess debt by $^1/_{20}$, this should be reduced to $^1/_{25}$, advocating a more gradual return to the debt target. These findings are in line with the recommendations of Bhandari et al. (2017), who concluded that fiscal policy should aim for a long-term debt target with gradual return. The impact of the lagged debt ratio on the adjustment term is not unambiguous, as it affects the equation through multiple components. However, when considering its individual influence, not interacted with other components, it implies a positive relationship between the debt ratio and the adjustment term. This positive relationship aligns with the requirement for debt sustainability as suggested by Bohn (1998). A full understanding of the influence of the debt ratio on the adjustment term requires further investigation. Finally, it is worth mentioning that the coefficient of the output gap is positive, yet fairly small. The countercyclicality evaluation measure reveals that the impact of the output gap is indeed small, as suggested by its coefficient. An ideal scenario features a significant positive coefficient for the output gap, indicating a countercyclical policy response. In this context, a positive term signifies that a positive output gap leads to a positive adjustment in the primary balance, which is achievable through increased revenue or decreased expenses. If the main objective is to promote counterclicality, it may be interesting to impose a restriction on θ_1, however, this is expected to deteriorate the volatility level.

Furthermore, optimising under various assumptions indicates that the size of the coefficients varies in response to changes in the economic environment. While under the baseline assumptions, only ⅕th of the expected change must be reversed, changing the assumptions leads to an increased need for compensation, making these scenarios more consistent with the findings of Barro (1979). Additionally, the impact of the output gap becomes stronger as well, which is also reflected in the countercyclical efficiency measure. These results suggest that under changing circumstances, except for a decrease in interest rate, it is beneficial to maintain a more countercylical fiscal policy. To conclude, the findings imply that if circumstances are more challenging (e.g., due to a larger difference between the interest rate and the growth rate or a deterioriated base path of the primary balance), a larger need for explicit policy intervention arises, as the majority of the coefficients increases. This is also expressed in the elevated volatility levels. Additionally, note that the impact of the change in assumptions does not seem to be linear and/or equal for the various changes in assumptions on growth, interest, and primary balance. This implies that it is essential to exercise prudence when expectations change. Also, further research in this field is recommended.

Hence, based on this analysis, it can be concluded that ideal fiscal policy should only partially counterbalance the expected shift, respond positively but moderately to the output gap, and be characterised by a gradual approach to a predetermined debt goal. Adjusting the neutral debt target was found to lead to an improved policy rule, characterised by a lower level of volatility and a lower risk of exploding debt. However, a DSA simulation of the optimised results indicated no improvement in the degree of countercylical efficiency. As such, it can be concluded that adjusting the neutral debt strategy with explicit debt goals and business cycle indicators leads to an improved fiscal policy regarding stability and the risk of default.

6.1 Limitations

This analysis attempted to replicate the real world as accurately as possible, however, limitations always remain. First, the effects of fiscal policy on the economy are highly uncertain and vary depending on the (economic) situation (as concluded by Katz and Bettendorf (2023)). Nevertheless, in the model, the fiscal multiplier is included as deterministic and constant. As multipliers were found to be higher in times of recessions and in response to restrictive policy changes, the multiplier used in this analysis is likely to underestimate the detrimental impact of contractions in recessions. Similarly, this analysis fails to differentiate the benefits of different investments. Second, the model does not incorporate the effects of hysteresis, which further contributes to a potential underestimation of the impact of fiscal consolidation during recessionary periods.

In addition, the current optimised fiscal policy rule function relies on the (long-term) expectation of interest and growth. While the proxy used in this model has demonstrated effectiveness, it may not be the optimal choice. The assumption that the growth potential remains constant over time in this model contradicts historical evidence, where expectations of economic growth have been known to change over time, consequently influencing the development of the debt ratio.

For future research, exploring the optimisation of nonlinear functions for discretionary policy could be interesting. In this analysis, the adjustment term of the primary balance is confined to a linear form. However, investigating interactions or nonlinear relationships will probably yield valuable insights. Additionally, the optimisation model exhibited a relatively long convergence time; this issue could potentially be mitigated by employing a different gradient estimator characterised by lower variance, such as the IPA estimator.

7 Conclusion

This thesis aimed to improve fiscal policy by extending on the debt sustainability analysis framework and introducing a novel approach to evaluate policy rules, with a focus on the following research question: *Does amending a neutral debt strategy with explicit debt goals and business cycle indicators produce a strategy that is more stable, countercyclical and characterised by a low risk of default?* The analysis tries to bridge the gap between theoretical welfare optimisation and empirical phenomena that impact fiscal policy, including business cycles and fiscal multipliers. The results of the analysis illustrated the potential for reforming fiscal policy by modifying straightforward rules drawn from optimisation outcomes found in existing literature, such as a neutral and explicit debt target.

The optimised fiscal policy rule showed a lower level of volatility, while keeping the debt ratio within the allowed debt space. The benchmark scenarios, which included ad hoc debt and deficit limitations, as well as a neutral and explicit debt goal, significantly decreased the risk of default; however, they concurrently introduced instability, particularly the ad hoc restrictions. The analysis concluded that optimal fiscal policy should revert part of the expected change in the debt ratio, react positively, yet moderately, to the state of the economy, and initiate a gradual return to a predetermined debt target. The level of reversion, countercyclical reaction, and the speed of return turned out to depend on the underlying assumptions of economic development. In general, it was found that more pessimistic assumptions require more significant policy adjustments.

Relating it back to the research question, it can be concluded that the adjusting straightforward rules obtained from the literature results in an improvement fiscal policy rule, compared to the benchmark scenarios. The modified policy was found to reduce volatility and effectively manage the risk of default. However, it does not promote countercylicality. Fortunately, it also avoids triggering procyclical policy.

For future research, it is interesting to refine the economic model used to model the development of the debt ratio. This analysis resorted to a relatively simple representation of the economy that did not account for all the minor details. Also, it might be interesting to look into other nonlinear function forms for the adjustment term, including other predictors for discretionary policy. Furthermore, the time horizon in this analysis is fixed at $T = 40$ years, whereas the true problem does not have a finite time horizon. Therefore, it may be interesting to look into a longer time-horizon, possibly even approaching the infinite time-horizon.

A Appendix: Evaluation of the neutral debt target proxy

As a shortcut for the expected value of $\frac{1+r_t}{1+g_t}$ conditional on time $t-1$, this analysis resorted to $\frac{1+\hat{r}_t}{1+\hat{g}_t}$, with $\hat{r}_t = r_{t-1}$ and $\hat{g}_t = g_t^p$. To evaluate this proxy, we will compare the expected value of $\frac{1+r_1}{1+g_1}$ obtained using Monte Carlo simulation with the value obtained using the proxy at $t = 1$. Given the endogenous relationships in the model, it is computationally too intense to evaluate the proxy in multiple years. However, the fact that each iteration is initiated with the same starting values allows us to capture the development to the next period from a fixed starting point. As such, the expected value at $t = 1$ can be obtained using the fact that

$$\frac{1}{N}\sum_{i=1}^{N}\frac{1+r_{i,1}}{1+g_{i,1}} \to \mathbb{E}\left[\frac{1+r_1}{1+g_1}\middle|\mathcal{F}_0\right], \qquad \text{for } N \to \infty, \tag{53}$$

with $r_{i,1}$ and $g_{i,1}$ the value of the r_1 and g_1 in the i-th iteration respectively. The expected value was found to be 0.9644 (after $N = 10,000$ iterations), while the proxy leads to 0.9703. This implies that the approximation error is rather small, namely less than 1%. However, the compound value of a small error can still have a significant impact.

B Appendix: Gradient Analysis

Figure 7 shows the distribution of the partial derivatives for $\theta_i, i = 0, ..., 3$, using $n = 300$ iterations of the gradient. It can be observed that the distributions of θ_0 and θ_3 are characterised by a much larger standard deviation, which may obstruct the convergence of the other coefficients. This is also in line with the partial derivatives derived using the IPA estimator.

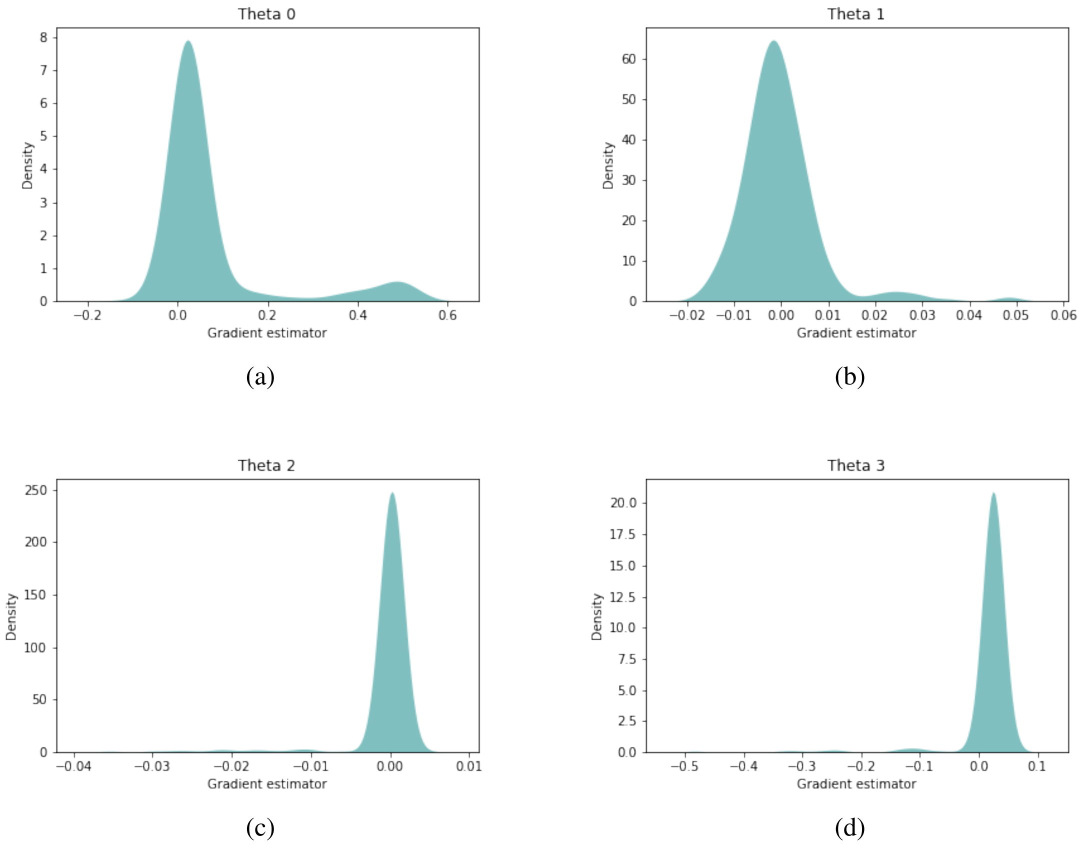


Figure 7: Distributions from the meta analysis

C Appendix: Distribution of the gradient norm

Figure 8 shows the distribution of the gradient norm, obtained using the Euclidean norm. The distribution is clearly skewed to the right and remains predominantly below 0.5. There are some outliers with values far above 0.5, as these values only disturb the optimisation process, it was chosen to clip the gradient at 0.5.

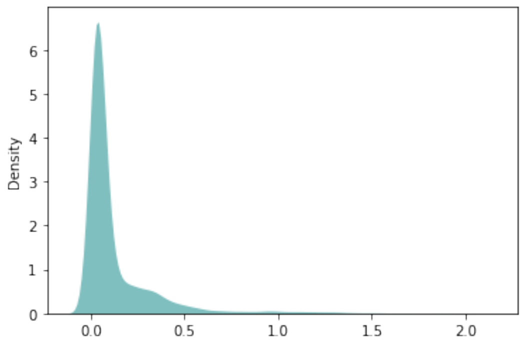
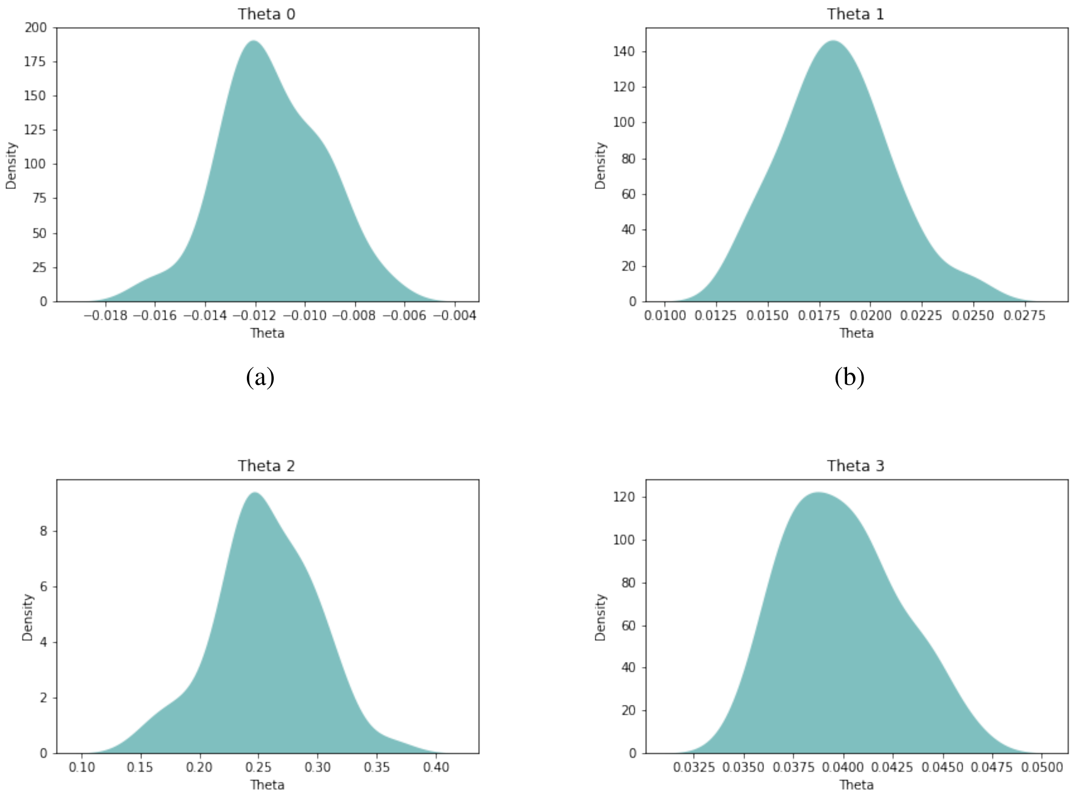


Figure 8: Histogram of the gradient norms

D Appendix: Meta analysis optimisation results

A meta analysis of the optimisation results shows the distribution of individual coefficients θ_i for $i = 0, ..., 3$. The optimisation process was run 50 times and in each run the average of the last 50 iterations was taken as an estimate of the coefficient. This has led to the distributions depicted in Figure 9. Using the Shapiro-Wilk test for normality, we fail to reject the null hypothesis of normality for all densities; see Table 5. These results are also confirmed using the Jarque-Bera test for normality. Moreover, the standard deviation indicates that the uncertainty in the estimate is relatively small.



(a) (b) (c) (d)

Figure 9: Distributions from the meta analysis

Table 5: Descriptive statistics

θ_i	**Mean**	**Std.**	**SW**	**JB**
0	-0.011	0.002	0.987 (0.862)	0.015 (0.992)
1	0.018	0.003	0.984 (0.713)	1.003 (0.606)
2	0.255	0.041	0.985 (0.765)	0.181 (0.914)
3	0.040	0.003	0.972 (0.2786)	2.031 (0.3622)

SW: Shapiro-Wilk test; JB = Jarque-Bera test; p-values in parentheses

References

Abdel-Kader, Khaled (2013). *What are structural policies?* Tech. rep. IMF Finance and Development.

Aiyagari, S Rao, Albert Marcet, Thomas J Sargent, and Juha Seppälä (2002). “Optimal taxation without state-contingent debt”. In: *Journal of Political Economy* 110.6, pp. 1220–1254.

Algemene Rekenkamer (2021). *Aandachtspunten bij de ontwerpbegroting 2022 van het Ministerie van Infrastructuur en Waterstaat*. Tech. rep. Algemene Rekenkamer.

Auerbach, Alan J and Yuriy Gorodnichenko (2012). “Measuring the output responses to fiscal policy”. In: *American Economic Journal: Economic Policy* 4.2, pp. 1–27.

Backus, David and John Driffill (1985). “Rational expectations and policy credibility following a change in regime”. In: *The Review of Economic Studies* 52.2, pp. 211–221.

Baldi, Guido and Karsten Staehr (2016). “The European debt crisis and fiscal reactions in Europe 2000–2014”. In: *International Economics and Economic Policy* 13, pp. 297–317.

Barnichon, Regis, Davide Debortoli, and Christian Matthes (2022). “Understanding the size of the government spending multiplier: It’s in the sign”. In: *The Review of Economic Studies* 89.1, pp. 87–117.

Barro, Robert J (1979). “On the Determination of the Public Debt”. In: *Journal of Political Economy* 87.5, pp. 940–971.

Batini, Nicoletta, Luc Eyraud, Lorenzo Forni, and Anke Weber (2014). *Fiscal Multipliers: Size, Determinants, and Use in Macroeconomic Projections*. Tech. rep. International Monetary Fund.

Berge, Travis, Maarten De Ridder, and Damjan Pfajfar (2021). “When is the fiscal multiplier high? A comparison of four business cycle phases”. In: *European Economic Review* 138, p. 103852.

Berti, Katia, Eugeniu Colesnic, Cyril Desponts, Stéphanie Pamies, and Etienne Sail (2016). “Fiscal Reaction Functions for European Union Countries”. In: *European Commission Discussion Paper* 28.

Bezemer, Dirk (2020). *Opinie: schulden zijn oké als ze geld opleveren*. `https://www.volkskrant.nl/columns-opinie/opinie-schulden-zijn-oke-als-ze-geld-opleveren~b5ca74aa/`. Accessed: 2023-09-19.

Bhandari, Anmol, David Evans, Mikhail Golosov, and Thomas J Sargent (2017). “Fiscal policy and debt management with incomplete markets”. In: *The Quarterly Journal of Economics* 132.2, pp. 617–663.

Blanchard, Olivier (2019). “Public debt and low interest rates”. In: *American Economic Review* 109.4, pp. 1197–1229.

Blanchard, Olivier, Alessia Amighini, and Francesco Giavazzi (2017). *Macroeconomics. A European Perspective*. Pearson.

Blanchard, Olivier, Alvaro Leandro, and Jeromin Zettelmeyer (2021). “Redesigning EU fiscal rules: From rules to standards”. In: *Economic Policy* 36.106, pp. 195–236.

Bohn, Henning (1998). “The behavior of US public debt and deficits”. In: *the Quarterly Journal of economics* 113.3, pp. 949–963.

Bouabdallah, Othman, Cristina D Checherita-Westphal, Thomas Warmedinger, Roberta De Stefani, Francesco Drudi, Ralph Setzer, and Andreas Westphal (2017). “Debt sustainability analysis for euro area sovereigns: a methodological framework”. In: *ECB Occasional Paper* 185.

Brainard, William C (1967). “Uncertainty and the Effectiveness of Policy”. In: *The American Economic Review* 57.2, pp. 411–425.

Braun, Miguel and Mariano Tommasi (2004). “Subnational fiscal rules: a game theoretic approach”. In: *Rules-Based Fiscal Policy in Emerging Markets: Background, Analysis, and Prospects*. Springer, pp. 183–197.

Caballero, Ricardo J, Emmanuel Farhi, and Pierre-Olivier Gourinchas (2017). "The safe assets shortage conundrum". In: *Journal of economic perspectives* 31.3, pp. 29–46.
Cacciatore, Matteo, Romain Duval, Davide Furceri, and Aleksandra Zdzienicka (2021). "Fiscal multipliers and job-protection regulation". In: *European Economic Review* 132, p. 103616.
CBS Statistics Netherlands (2023). *Inflation rate down to 4.6 percent in July*. https://www.cbs.nl/en-gb/news/2023/32/inflation-rate-down-to-4-6-percent-in-july. Accessed: 2023-15-08.
Cecchetti, Stephen G (1995). "Inflation indicators and inflation policy". In: *NBER macroeconomics annual* 10, pp. 189–219.
Cecchetti, Stephen G, Rita S. Chu, and Charles Steindel (2000). "The unreliability of inflation indicators". In: *Current Issues in Economics and Finance* 6.Apr, p. 4.
Centraal Bureau voor de Statistiek (2022). *Dashboard Overheidsfinanciën*. https://www.cbs.nl/nl-nl/visualisaties/dashboard-overheidsfinancien. Accessed: 2023-22-08.
Cerra, Valerie, Antonio Fatás, and Sweta C Saxena (2023). "Hysteresis and business cycles". In: *Journal of Economic Literature* 61.1, pp. 181–225.
Checherita-Westphal, Cristina and Václav Žd'árek (2017). *Fiscal reaction function and fiscal fatigue: evidence for the euro area*. Working Paper 2036. European Central Bank.
CPB (n.d.). *Analyse coalitieakkoord 2022–2025*. https://open.overheid.nl/documenten/ronl-3074765a-dad6-4d99-a46b-79a47a250af4/pdf. Accessed: 2023-09-02.
DeLong, J Bradford and Laura D Tyson (2013). *Discretionary Fiscal Policy as a Stabilization Policy Tool*. Tech. rep. International Monetary Fund.
Dicou, Diederik (2022). *Antwoorden op veelgestelde vragen over de CPB-ramingen*. Tech. rep. CPB.
DNB (n.d.). *The ECB's monetary policy*. https://www.dnb.nl/en/the-euro-and-europe/the-ecb-s-monetary-policy/. Accessed: 2023-12-09.
European Commission (2017). *Sustainability of public finances*. https://commission.europa.eu/system/files/2020-06/european-semester_thematic-factsheet_public-finance-sustainability_en_0.pdf. Accessed: 2023-05-08.
— (2023). *Stability and Growth Pact*. https://economy-finance.ec.europa.eu/economic-and-fiscal-governance/stability-and-growth-pact_en. Accessed: 2023-18-08.
Fotiou, Alexandra (2022). "Non-linearities in fiscal policy: The role of debt". In: *European Economic Review* 150, p. 104212.
Fournier, Jean-Marc and Falilou Fall (2015). *Limits to government debt sustainability*. Tech. rep. OECD.
Ghosh, Atish R, Jun I Kim, Enrique G Mendoza, Jonathan D Ostry, and Mahvash S Qureshi (2013). "Fiscal fatigue, fiscal space and debt sustainability in advanced economies". In: *The Economic Journal* 123.566, F4–F30.
Ghosh, Atish R, Jonathan D Ostry, and Mahvash S Qureshi (2013). "Fiscal space and sovereign risk pricing in a currency union". In: *Journal of International Money and finance* 34, pp. 131–163.
Guzman, Martin and Daniel Heymann (2015). "The IMF debt sustainability analysis: issues and problems". In: *Journal of Globalization and Development* 6.2, pp. 387–404.
Hamilton, James D and Marjorie A Flavin (1985). *On the Limitations of Government Borrowing: A Framework for Empirical Testing*. Working Paper 1632. National Bureau of Economic Research. DOI: 10.3386/w1632. URL: http://www.nber.org/papers/w1632.

Hartmann, Philipp and Frank Smets (2018). *The first twenty years of the European Central Bank: monetary policy*. Working paper 2219. European Central Bank.

Heimberger, Philip (2023). *Debt sustainability analysis as an anchor in EU fiscal rules*. Tech. rep. European Parliament.

Heller, Peter (2005). *Fiscal Space: What*. Tech. rep. IMF Finance and Development, pp. 32–33.

Homan, Eitel and Wim Suyker (2015). *Hoe anticyclisch is het Nederlandse discretionaire begrotingsbeleid*. Tech. rep. CPB Netherlands Bureau for Economic Policy Analysis.

Howitt, Peter (2010). "Endogenous growth theory". In: *Economic growth*. Springer, pp. 68–73.

Ilzetzki, Ethan, Enrique G Mendoza, and Carlos A Végh (2013). "How big (small?) are fiscal multipliers?" In: *Journal of monetary economics* 60.2, pp. 239–254.

IMF (2022). *Staff Guidance Note on the Sovereign Risk and Debt Sustainability Framework for Market Access Countries*. Tech. rep. International Monetary Fund.

Jacobs, Bas (2015). *Overheid heeft met falend begrotingsbeleid een derde van de Grote Recessie veroorzaakt*. https://esb.nu/overheid-heeft-met-falend-begrotingsbeleid-een-derde-van-de-grote-recessie-veroorzaakt/. Accessed: 2023-08-18.

— (2020). "Beleidsfouten dreigen door verkeerde conclusies uit houdbaarheidssommen". In: *Economische Statistische Berichten* 105.4781, pp. 028–031.

Jahan, Sarwat and Ahmed Saber Mahmud (2013). "What is the output gap". In: *Finance & Development* 50.3, pp. 38–39.

Katz, Matthijs and Leon Bettendorf (2023). *Nieuwe inzichten over multipliers overheidsbestedingen*. Tech. rep. CPB.

Ketkar, Nikhil (2017). "Stochastic gradient descent". In: *Deep learning with Python: A hands-on introduction*, pp. 113–132.

Khan, Mehreen (2019). *Netherlands rolls out economic stimulus plan as downturn looms*. https://www.ft.com/content/d45d8378-d962-11e9-8f9b-77216ebe1f17. Accessed: 2023-22-08.

Koop, Gary and Dimitris Korobilis (2012). "Forecasting inflation using dynamic model averaging". In: *International Economic Review* 53.3, pp. 867–886.

Kremers, Jeroen JM (1989). "US federal indebtedness and the conduct of fiscal policy". In: *Journal of Monetary economics* 23.2, pp. 219–238.

Leeuw, Daan de and Krista Bruns (2022). "Rutte IV intensiveert historisch veel". In: *Economische Statistische Berichten* 107.4807, p. 104.

Lucas Jr, Robert E and Nancy L Stokey (1983). "Optimal fiscal and monetary policy in an economy without capital". In: *Journal of monetary Economics* 12.1, pp. 55–93.

Lukkezen, Jasper (2021). "Regie broodnodig op de woningmarkt, geld in mindere mate". In: *Economische Statistische Berichten* 107.4805, pp. 028–029.

Lukkezen, Jasper, Hugo Rojas-Romagosa, et al. (2012). *When is debt sustainable?* Tech. rep. CPB Netherlands Bureau for Economic Policy Analysis.

Lukkezen, Jasper and Hugo Rojas-Romagosa (2013). "Stochastic debt sustainability indicators 1". In: *Revue de l'OFCE* 1, pp. 97–121.

Mauro, Mr Paolo and Jing Zhou (2020). *r minus g negative: Can We Sleep More Soundly?* Working Paper 2020/052. International Monetary Fund.

Mohl, Philipp, Gilles Mourre, Klara Stovicek, et al. (2019). *Automatic fiscal stabilisers in the EU: Size and effectiveness*. Tech. rep. Directorate General Economic and Financial Affairs (DG ECFIN), European Commission.

Moreno Badia, Marialuz, Paulo A Medas, Pranav Gupta, and Yuan Xiang (2020). *Debt is not Free*. Working Paper 2020/001. International Monetary Fund.

Mourre, Gilles, Caterina Astarita, Savina Princen, et al. (2014). *Adjusting the budget balance for the business cycle: the EU methodology*. Tech. rep. Directorate General Economic and Financial Affairs (DG ECFIN), European Commission.

Ramsey, Frank Plumpton (1928). "A mathematical theory of saving". In: *The economic journal* 38.152, pp. 543–559.

Rijksoverheid (n.d.). *Rijksjaarverslag: inkomsten en uitgaven*. `https://www.rijksoverheid.nl/onderwerpen/verantwoordingsdag/rijksjaarverslag-inkomsten-en-uitgaven`. Accessed: 2023-11-09.

Romer, Paul M (1994). "The origins of endogenous growth". In: *Journal of Economic perspectives* 8.1, pp. 3–22.

Saulo, Helton, Leandro C Rêgo, and Jose A Divino (2013). "Fiscal and monetary policy interactions: a game theory approach". In: *Annals of Operations Research* 206, pp. 341–366.

Shaw, Graham K (1992). "Policy implications of endogenous growth theory". In: *The Economic Journal* 102.412, pp. 611–621.

Stock, James H and Mark W Watson (1999). "Forecasting inflation". In: *Journal of monetary economics* 44.2, pp. 293–335.

Studiegroep Begrotingsruimte (2020). *Koers bepalen. Kiezen in tijden van budgettaire krapte*. Ministerie van Financiën.

Teulings, Coen (2016). "Are Bubbles Bad? Is a Higher Debt Target for the Eurozone Desirable?" In: *CESifo Economic Studies* 62.2, pp. 197–209.

— (2020a). "De private sector verlangt een hogere staatsschuld en dat levert oo nog wat op". In: *Economische Statistische Berichten* 105.4788, pp. 364–367.

— (2020b). "De private sector verlangt een hogere staatsschuld en dat levert ook nog wat op". In: *Economische Statistische Berichten*, pp. 364–367.

Vázquez-Abad, Felisa and Bernd Heidergott (2023). *Optimization and Learning via Stochastic Gradient Search*.

Vries, Gert Jan de (2020). *Niemand wilde effect bezuinigingen bij Belastingdienst onder ogen zien*. `https://www.trouw.nl/opinie/niemand-wilde-effect-bezuinigingen-bij-belastingdienst-onder-ogen-zien~bfd29de4/`. Accessed: 2023-20-08.

Weichenrieder, Alfons J and Jochen Zimmer (2014). "Euro membership and fiscal reaction functions". In: *International Tax and Public Finance* 21, pp. 598–613.

Wijnbergen, Sweder van (2021). *Problemen bij de Belastingdienst: beschuldig niet de ambtenaren*. `https://www.nrc.nl/nieuws/2021/10/15/problemen-bij-de-belastingdienst-beschuldig-niet-de-ambtenaren-a4062021`. Accessed: 2023-20-08.

Willems, Tim and Jeromin Zettelmeyer (2022). "Sovereign debt sustainability and central bank credibility". In: *Annual Review of Financial Economics* 14, pp. 75–93.

Zedda, Stefano, Matteo Salto, AA.VV, and Lucio Pench (2011). "Public finances in EMU - 2011". In: *European Economy* 3. DOI: `10.2765/15327`.